Appendix D: Trigonometry

1 Angles, radians, degrees

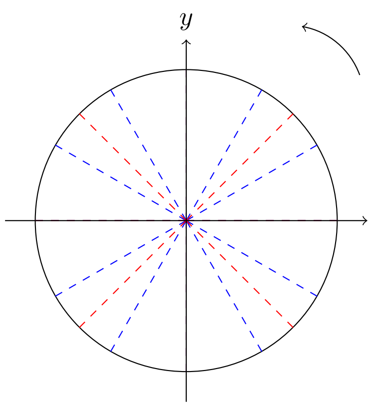
Angles can be measured in radians (rad) or in degrees (°) with the relation

$$2\pi = 360°.$$

$$\begin{aligned} 1° &= \\ 1 \text{ rad} &= \end{aligned}$$

Exercise 1. Complete the following table and place those angles on the trigonometric circle.

Degree	0°	30°			90°	120°	135°			270°	
Radians			$\frac{\pi}{4}$	$\frac{\pi}{3}$				$\frac{5\pi}{6}$	π		2π



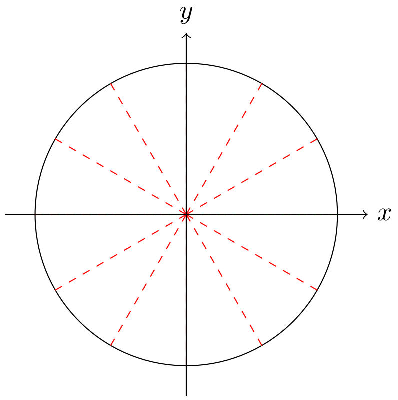
Remark:

Exercise 2. Plot on the trigonometric circle the angles $\frac{-\pi}{4}$, $\frac{-2\pi}{3}$.

Exercise 3. Convert 72° in radians.

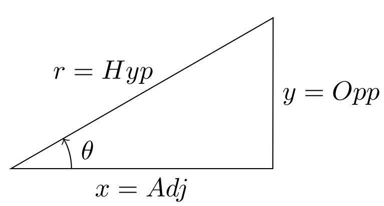
Exercise 4. Convert $\frac{7\pi}{10}$ in degrees.

Exercise 5. Let $\mathcal{C}$ be a circle of radius 4cm. $\theta = \frac{4\pi}{3}$ be a central angle.
What is the length of the arc subtended by θ?



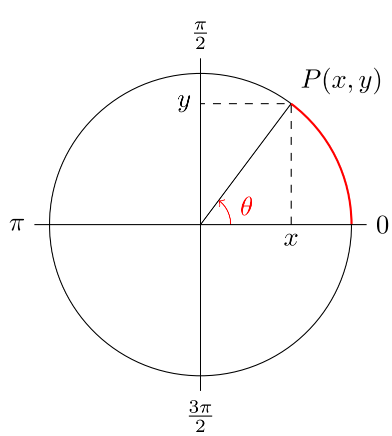
2 Trigonometric Functions

Definition: Given the following right triangle



$\cos\theta =$	$\sec\theta =$
$\sin\theta =$	$\csc\theta =$
$\tan\theta =$	$\cot\theta =$

Definition: Let $P(x, y)$ be a point of the trigonometric circle. Let θ be the angle $([0x), [O, P))$.



Then $\cos\theta = x \quad \sin\theta = y$

Remark:

Exercise 6. Given $\sin\theta = \frac{2}{5}$ and $0 \leqslant \theta \leqslant \frac{\pi}{2}$,
find $\cos\theta$, $\tan\theta$, $\sec\theta$, $\csc\theta$, $\operatorname{cotan}\theta$.

Exercise 7. (Spring 2012) Given $\sin x = \frac{3}{4}$. x lies in Quadrant II, What is $\tan x$?

Formulae:

$\theta(radian)$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$
$\cos\theta$										
$\sin\theta$										
$\tan\theta$										

Exercise 8. (Spring 2012) Evaluate $\cos\left(\frac{7\pi}{6}\right)$.

3 Trigonometric Identities

Trigonometric identities:

$$\sec\theta = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{1}{\sin\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$$

$$\begin{aligned} \sin(x+y) &= \sin x\cos y + \cos x\sin y & \cos(x+y) &= \cos x\cos y - \sin x\sin y \\ \sin(x-y) &= \sin x\cos y - \cos x\sin y & \cos(x-y) &= \cos x\cos y + \sin x\sin y \\ \sin(2x) &= 2\sin x\cos x & \cos(2x) &= \cos^2 x - \sin^2 x \\ & & &= 2\cos^2 x - 1 \\ & & &= 1 - 2\sin^2 x \end{aligned}$$

Exercise 9. Prove that $\cos\left(\frac{\pi}{2} - x\right) = \sin x$ for any x.

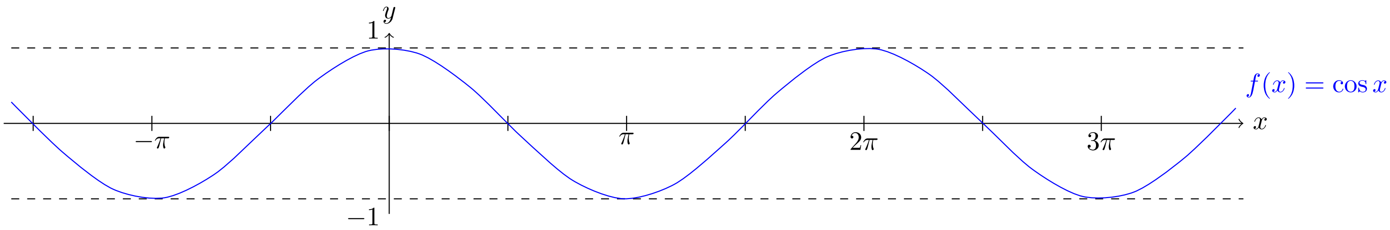
Exercise 10. Prove that $\tan x + \tan y = \dfrac{\sin(x+y)}{\cos x\cos y}$.

Exercise 11. Solve for x in the interval $[0, 3\pi]$, the equation

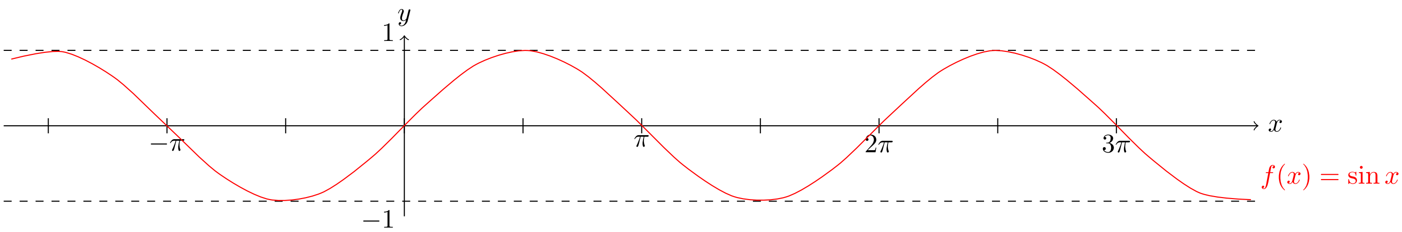
$$2\cos x + \sin 2x = 0$$

4 Graph of the trigonometric functions

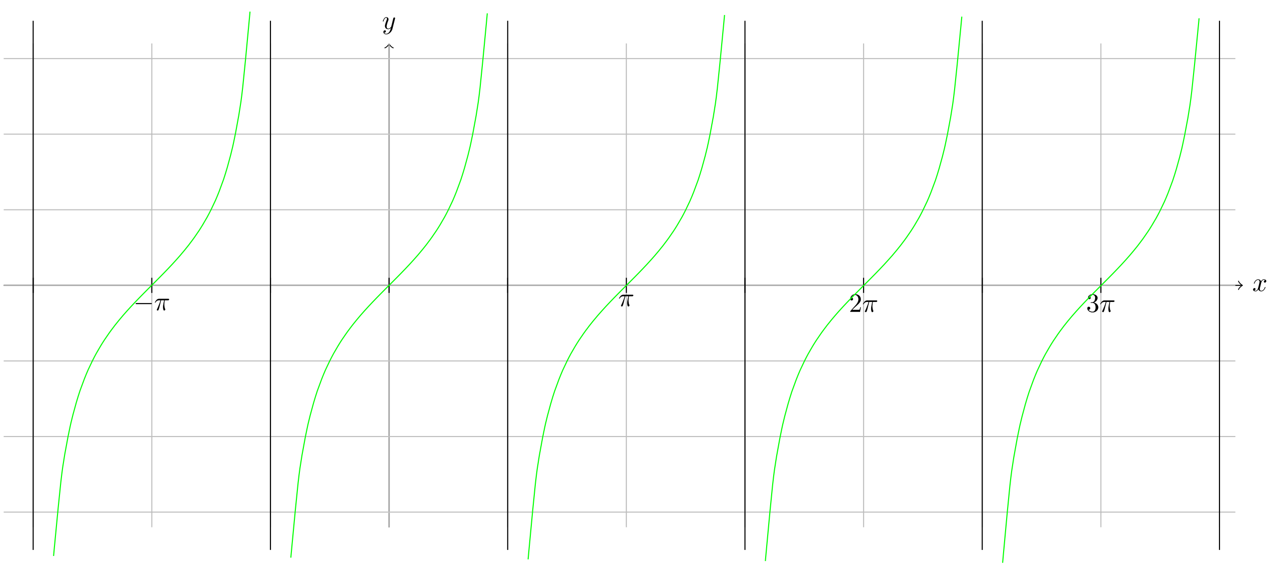
4.1 Cosinus



4.2 Sinus



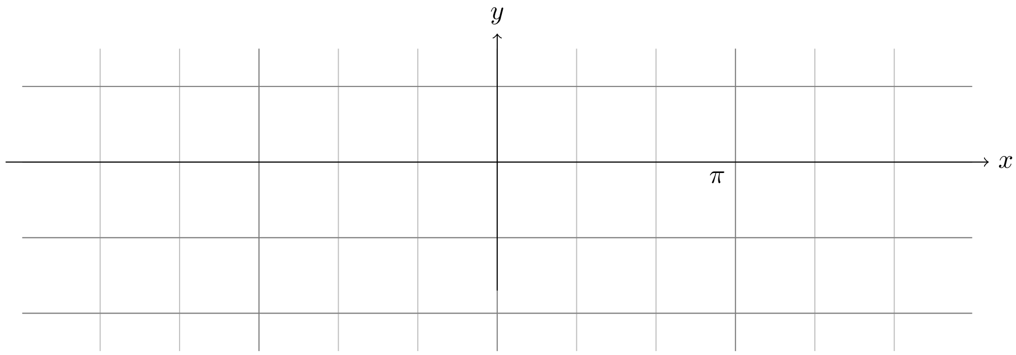
4.3 Tangent



See your book for the graphs of $\cot x$, $\sec x$, $\csc x$...

Exercise 12. Graph the following functions by applying transformations

- $y = \cos\left(x + \frac{\pi}{3}\right)$.



- $y = \tan\left(x - \frac{\pi}{4}\right) + 2.$