Exercise 3. (58p193) Find the x-coordinates of all the points on the curve

$$y = \sin 2x - 2\sin x$$

at which the tangent line is horizontal.

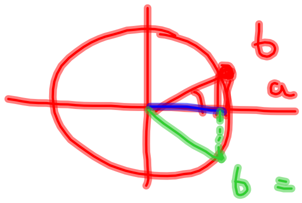
Find x such that $y'(x) = 0$

$$y'(x) = \cos(2x) \times (2x)' - 2\cos x$$

$$= 2\cos 2x - 2\cos x = 0$$

$k \in \mathbb{Z}$

$\cos a = \cos b$



$b = a + 2k\pi$

$b = -a + 2k\pi$

$\cos 2x = \cos x$

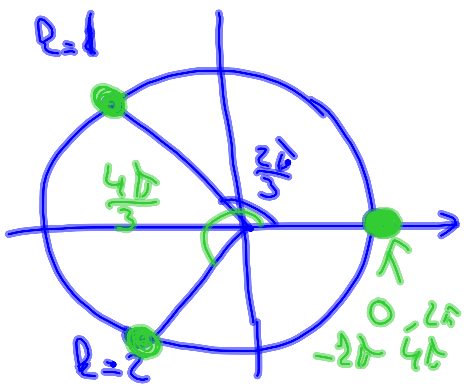
$$\begin{cases} 2x = x + 2k\pi \\ \text{or} \\ 2x = -x + 2k\pi \end{cases}$$

$$\begin{cases} x = 0 + 2k\pi \\ 3x = 2k\pi \end{cases} \quad k \in \mathbb{Z}$$

$\cos a = \cos b$

$$\begin{cases} a = b + 2k\pi \\ a = -b + 2k\pi \end{cases}$$

$$\begin{cases} x = 0 + 2k\pi \\ x = \dfrac{2k\pi}{3} \end{cases} \quad k \in \mathbb{Z}$$



Exercise 4. Suppose that $u(x) = f(g(x))$ and

$$f(0) = 3, \quad f'(0) = 2, \quad f(1) = 5, \quad f'(1) = -4, \quad g(0) = 1, \quad g'(0) = 7.$$

Find $u'(0)$

$u'(x) = f'(g(x)) \times g'(x)$

$x=1$ $u'(0) = f'(g(0)) \times g'(0)$ $g(0) = 1$

$= f'(1) \times 7$

$= (-4) \times 7 = -28$

Section 3.6

Exercise 5. Find an equation to the tangent line to the curve $y = \tan(\pi x^3/4)$ at the point $(1, 1)$. ~~Find $y'(0)$.~~

- slope of the tangent line

$x = 1$
$y = 1$

slope $= y'(1)$

$$y(x) = \tan\left(\frac{\pi x^3}{4}\right)$$

$$y'(x) = \tan'\left(\frac{\pi x^3}{4}\right) \times \left(\frac{\pi x^3}{4}\right)'$$

$$y'(x) = \sec^2\left(\frac{\pi x^3}{4}\right) \times \frac{3\pi x^2}{4}$$

$$y'(1) = \sec^2\left(\frac{\pi}{4}\right) \times \frac{3\pi}{4}$$

$$= \frac{1}{\left(\cos\left(\frac{\pi}{4}\right)\right)^2} \times \frac{3\pi}{4}$$

$$= \frac{1}{\left(\frac{1}{\sqrt{2}}\right)^2} \times \frac{3\pi}{4}$$

$$= \frac{1}{\frac{1}{2}} \times \frac{3\pi}{4} = \boxed{\frac{3\pi}{2}}$$

slope of tangent line at $x=1$

Equation tangent line

slope point equation

$$\boxed{y - 1 = \frac{3\pi}{2}(x - 1)}$$

slope

value of y at the point

value of x at the point

3.6

Exercise 1. Find an equation of the tangent line to the curve $y^2 = x^3(2-x)$ at the point $(1,1)$.

$x=1$, $y=1$

We need $y'(1)$.

* $y = \sqrt{x^3(2-x)} = (x^3(2-x))^{1/2}$

...

* y is a mysterious function

$y(x)$

$$y^2(x) = x^3(2-x)$$

2 functions of x equal
⇒ they have the same derivative

$$2y(x)\,y'(x) = 3x^2(2-x) + x^3(-1)$$

$$2y\frac{dy}{dx}$$

$$2y\frac{dy}{dx} = 6x^2 - 3x^3 - x^3 = 6x^2 - 4x^3$$

if $x=1$, $y=1$: $$2\frac{dy}{dx} = 6 - 4 = 2$$

$$\left.\frac{dy}{dx}\right|_{x=1,\,y=1} = 1$$

slope of tangent to the curve

$$y - 1 = 1(x-1)$$

$$y = x$$

Exercise 2. Find $\frac{dy}{dx}$ using the implicit differentiation of

1. $y^5 + 3x^2y^3 + 5x^6 = 8.$

$y(x)$

$$(y(x))^5 + 3x^2(y(x))^3 + 5x^6 = 8$$

differentiate the equation with respect to x

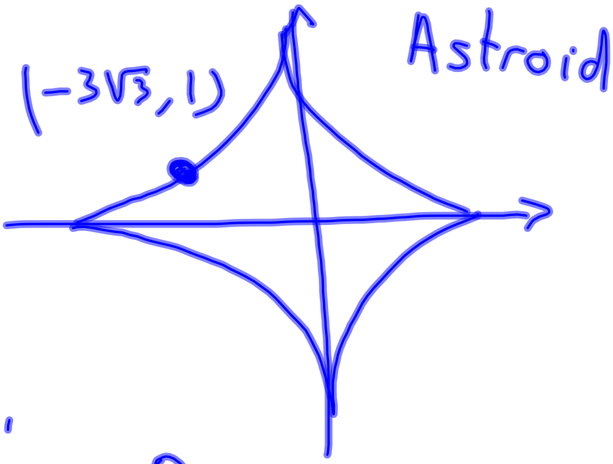
$$5(y(x))^4 \frac{dy}{dx} + 3 \times 2x\, y(x)^3 + 3x^2 \times 3y(x)^2 \frac{dy}{dx} + 5 \times 6x^5 = 0$$

$$5y^4 \frac{dy}{dx} + 6xy^3 + 9x^2y^2 \frac{dy}{dx} + 30x^5 = 0$$

$$(5y^4 + 9x^2y^2)\frac{dy}{dx} + (6xy^3 + 30x^5) = 0$$

$$\frac{dy}{dx} = -\frac{6xy^3 + 30x^5}{5y^4 + 9x^2y^2}$$

2. $x^{2/3} + y^{2/3} = 4$ at $(-3\sqrt{3}, 1)$.



$y(x)$

$x^{2/3} + (y(x))^{2/3} = 4$

$\frac{2}{3} x^{2/3 - 1} + \frac{2}{3} (y(x))^{2/3-1} \frac{dy}{dx} = 0$

$\frac{2}{3\sqrt[3]{x}} + \frac{2}{3\sqrt[3]{y}} \frac{dy}{dx} = 0$

$\left.\frac{dy}{dx}\right|_{(x,y)} = -\frac{\frac{2}{3\sqrt[3]{x}}}{\frac{2}{3}\sqrt[3]{y}} = \frac{\frac{1}{\sqrt[3]{x}}}{\frac{1}{\sqrt[3]{y}}}$

$= -\frac{\sqrt[3]{y}}{\sqrt[3]{x}}$

$x = -3\sqrt{3}$

$x = -(\sqrt{3})^2 \times \sqrt{3}$

$x = -(\sqrt{3})^3$

$\sqrt[3]{x} = \sqrt[3]{-(\sqrt{3})^3}$

$\sqrt[3]{x} = -\sqrt{3}$

$\sqrt[3]{y} = 1$

$\left.\frac{dy}{dx}\right|_{(-3\sqrt{3},1)} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

3. $\cos(x-y) = y\sin 3x - x\sin y.$

$\frac{dy}{dx}$ $\quad y(x)$

$$\cos(x - y(x)) = y(x)\sin(3x) - x\sin(y(x))$$

$$-\sin(x-y)\times\left(1-\frac{dy}{dx}\right) = \frac{dy}{dx}\sin 3x + y\cos(3x)\times 3 - \sin(y(x)) - x\cos(y(x))\frac{dy}{dx}$$

$$-\sin(x-y) + \frac{dy}{dx}\sin(x-y) = \frac{dy}{dx}(\sin 3x - x\cos y) + 3y\cos 3x - \sin(y)$$

$$\frac{dy}{dx}(\sin(x-y) - \sin 3x + x\cos y) = 3y\cos 3x - \sin y + \sin(x-y)$$

$$\frac{dy}{dx} = \frac{3y\cos 3x - \sin y + \sin(x-y)}{\sin(x-y) - \sin(3x) + x\cos y}$$

$\frac{dy}{dx}g(x) = f(x)$ ← each time equation of this form with f and g more of less complex.

$$\frac{dy}{dx} = \frac{f(x)}{g(x)}$$

4. $\frac{y}{y-3} = \sqrt{x+y}$

$$\frac{y(x)}{y(x)-3} = \left(x + y(x)\right)^{1/2}$$

$$\frac{\frac{dy}{dx}(y-3) - \frac{dy}{dx}(y)}{(y-3)^2} = \frac{1}{2}(x+y)^{-\frac{1}{2}} \times \left(1 + \frac{dy}{dx}\right)$$

$$\frac{\frac{dy}{dx}(y-3-y)}{(y-3)^2} = \frac{1}{2\sqrt{x+y}} + \frac{dy}{dx} \times \frac{1}{2\sqrt{x+y}}$$

$$\frac{dy}{dx}\left(\frac{-3}{(y-3)^2} - \frac{1}{2\sqrt{x+y}}\right) = \frac{1}{2\sqrt{x+y}}$$

$$\frac{dy}{dx} = \frac{\frac{1}{2\sqrt{x+y}}}{\left(\frac{-3}{(y-3)^2} - \frac{1}{2\sqrt{x+y}}\right)}$$

Exercise 3. (43p199) Find all points on the curve

$$x^2y^2 + xy = 2$$

where the slope is -1.

Find (x, y) such that $\begin{cases} y'(x) = -1 & \leftarrow \text{slope} = -1 \\ x^2y^2 + xy = 2 & \leftarrow \text{on the curve} \end{cases}$

$y'(x) = \frac{dy}{dx}$

$2xy^2 + 2x^2y\frac{dy}{dx} + y + x\frac{dy}{dx} = 0$ ⚠

$\frac{dy}{dx}(2x^2y + x) = -y - 2xy^2$

$\frac{dy}{dx} = \frac{-y - 2xy^2}{2x^2y + x} = -1$

$\begin{cases} y + 2xy^2 = 2x^2y + x & \leftarrow \text{slope} = -1 \\ x^2y^2 + xy = 2 & \leftarrow \text{on the curve} \end{cases}$

(2) $(xy)^2 + (xy) - 2 = 0$

(2) xy is solution to $X^2 + X - 2 = 0$

$(X+2)(X-1) = 0$

$xy = -2$ or $xy = 1$

* $\boxed{xy = -2}$

$y + 2y(xy) = 2(xy)x + x$

$y - 4y = -4x + x$

$-3y = -3x$ $\quad x = y \quad xy = -2$

$x = y \quad x^2 = -2$ impossible

No solution in this case

* $xy = 1$

$y + 2y(xy) = 2(xy)x + x$

$3y = 3x$

$y = x$ and $xy = 1$ $\quad x^2 = 1$, $y = x$

$y = x = 1$ or $y = x = -1$

2 solutions $\boxed{\begin{array}{l} x = 1 = y \\ x = -1 = y \end{array}}$ $\quad (1, 1)$, $(-1, -1)$

Exercise 4. (20p198) If

$$(g(x))^2 + 12x = x^2 g(x) \quad \text{and} \quad g(4) = 12,$$

find $g'(4)$.

Definition: Two curves are called orthoginal if at each point of intersection their tangent lines are perpendicular.

Exercise 5. Show that the given curves are orthogonal

1.
$$2x^2 + y^2 = 3, \quad x = y^2$$

2.

$$x^2 - y^2 = 5, \quad 4x^2 + 9y^2 = 72$$

Engineering Mathematics I
Math 151, Fall 2012

Texas A&M University

Section 3.7, derivatives of vector functions

Exercise 1. Sketch the curves with the given vector equation

$$\overrightarrow{r}(t) = \langle t^2 + 1, 2t \rangle$$

Indicate the direction in which t increases.

Exercise 2. Find the domain and derivative of the vector function

$$\overrightarrow{r}(t) = \langle \sqrt{t-4}, \frac{t^2}{t^2-1} \rangle$$

Exercise 3. Find a tangent vector of unit length at $t = \pi/4$ of $\overrightarrow{r}(t) = \langle t\cos t, t\sin t\rangle$.

Exercise 4. The vector function

$$\overrightarrow{r}(t) = \langle 4\cos t, 3\sin t\rangle$$

represents the position of a particle at time t. Find the velocity and speed at the point $t = \pi/3$.

Exercise 5. At what point do the curves traced by $\vec{r}_1(t) = \langle 1-t, 3+t^2 \rangle$ and $\vec{r}_2(s) = \langle s-2, s^2 \rangle$ intersect? Find their angle of intersection correct to the nearest degree.