Texas A&M University

Quiz 2 – Keys

1. Let

$$f(x) = \begin{cases} 2x+3 & \text{if } x < -1 \\ x^2 & \text{if } -1 \leqslant x \leqslant 1 \\ 3x-1 & \text{if } 1 < x \end{cases}$$

Answer.

$$C1: \lim_{x\to -1^+} f(x) = 1 \qquad \lim_{x\to -1^-} f(x) = 1 \qquad \lim_{x\to -1} f(x) = 1 \qquad f(-1) = 1$$

$$C2: \lim_{x\to 1^+} f(x) = 2 \qquad \lim_{x\to 1^-} f(x) = 1 \qquad \lim_{x\to 1} f(x) = DNE \qquad f(1) = 1$$

□

2. Find and justify

$$C1: \lim_{x\to 0} x^4 \sin\left(\frac{1}{x}\right)$$

$$C2: \lim_{x\to 0} x^6 \sin\left(\frac{1}{x}\right)$$

Answer.

$$-1 \leqslant \sin\left(\frac{1}{x}\right) \leqslant 1 \qquad\qquad -1 \leqslant \sin\left(\frac{1}{x}\right) \leqslant 1$$

$$-x^4 \leqslant x^4 \sin\left(\frac{1}{x}\right) \leqslant x^4 \qquad\qquad -x^6 \leqslant x^6 \sin\left(\frac{1}{x}\right) \leqslant x^6$$

$$\lim_{x\to 0} -x^4 = \lim_{x\to 0} x^4 = 0 \qquad\qquad \lim_{x\to 0} -x^6 = \lim_{x\to 0} x^6 = 0$$

`Therefore, by the Squeeze Theorem,`

$$\boxed{\lim_{x\to 0} x^4 \sin\left(\frac{1}{x}\right) = 0 \qquad \lim_{x\to 0} x^6 \sin\left(\frac{1}{x}\right) = 0}$$