

## Sections 4.5, 4.6, 4.8

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**Exercise 1.** A bacteria culture starts with 3000 bacteria and the population triples every 40 minutes.

1. Find an expression for the number of bacteria after  $t$  hours.
2. Find the number of bacteria after 30 minutes.
3. When will the population reach 20,000?

**Exercise 2.** A bacteria culture starts with 100 bacteria. After 3 hours, there are 8000 bacteria.

1. Find an expression for the number of bacteria after  $t$  hours.
2. Find the number of bacteria after 4 hours.
3. When will the population reach 200,000 bacteria?

**Exercise 3.** Iodine-131 has a half life of 8 days.

1. If a sample has a mass of 50mg, find the formula for the mass that remains after  $t$  days.
2. Find the mass that remains after 80 days.
3. How long would it take for the sample to decay to 10mg?

**Exercise 4.** A thermometer is taken from a room where the temperature is  $25^\circ$  to the outdoors, where the temperature is  $5^\circ$ . After 1 minute, the thermometer reads  $15^\circ$ , use the Newton's Law of Cooling to find

1. What will the reading on the thermometer be after one more minutes?
2. When will the thermometer read  $6^\circ$ ?

**Exercise 5.** How long will it take for a \$10,000 investment to triple if the interest rate is 5% invested weekly?

Same question if the investment is compounded continuously at a rate of 5%

**Exercise 6.** Find the exact value of

1.  $\text{Arctan}(-1)$
2.  $\cos(\text{Arctan } 4)$
3.  $\tan(\text{Arcsin}(\frac{2}{3}))$
4.  $\sec(\text{Arctan } 2)$ .
5.  $\text{Arccos}\left(\cos\left(\frac{-\pi}{3}\right)\right)$
6.  $\text{Arcsin}\left(\sin\left(\frac{1991\pi}{5}\right)\right)$

7.  $\text{Arctan} \tan \left( \frac{4\pi}{7} \right)$

**Exercise 7.** Find the derivative of

1.  $f(x) = \text{Arctan}(\text{Arcsin} \sqrt{x})$ . What is the domain of  $f$ , of  $f'$ ?
2.  $g(x) = x \text{Arccos} (x - \sqrt{1 - x^2})$ .
3.  $h(x) = \text{Arctan} (x - \sqrt{1 + x^2})$
4.  $j(x) = \text{Arcsin}(x) \ln(x)$  What is the domain of  $j$ , of  $j'$ ?

**Exercise 8.** Find an equation of the tangent line to the curve  $y = x + \text{Arctan} y$  at the point  $\left( \frac{\pi}{4} - 1, \frac{\pi}{4} \right)$

**Exercise 9.** Find the limits

1.  $\lim_{x \rightarrow \infty} \text{Arcsin} \left( \frac{x + 3}{2x - 5} \right)$ .
2.  $\lim_{x \rightarrow 0^+} \text{Arctan} \left( \frac{1}{\sin(x)} \right)$ .
3.  $\lim_{x \rightarrow 1^-} \text{Arcsin} (\text{Arccos} x)$ .
4.  $\lim_{x \rightarrow 3} \text{Arccos} \left( -e^{x^2 - 5x + 6} \right)$ .

**Exercise 10.** Prove that for any  $x \in [0, 1)$  and any  $y \in [0, 1)$ ,

$$\text{Arctan}(x) + \text{Arctan}(y) = \text{Arctan} \left( \frac{x + y}{1 - xy} \right)$$

**Exercise 11.** Find the limits of

1.  $\lim_{x \rightarrow \pi^-} \frac{\tan x}{1 - \cos x}$ .
2.  $\lim_{x \rightarrow \infty} \text{Arctan} \left( \frac{\sqrt{x}}{\ln x} \right)$ .
3.  $\lim_{x \rightarrow 0} \frac{\text{Arcsin} x - x}{x^3}$ .
4.  $\lim_{x \rightarrow 0} \frac{\text{Arcsin} x - 3x}{\text{Arctan} x + 3x}$ .
5.  $\lim_{x \rightarrow \infty} \frac{\ln(1 + e^{3x})}{5x}$ .
6.  $\lim_{x \rightarrow 0} (1 - \ln(x))^{\sin x}$ .
7.  $\lim_{x \rightarrow \infty} \left( \frac{2x + 5}{2x - 7} \right)^{2x+3}$ .
8.  $\lim_{x \rightarrow \infty} \left( 1 + \frac{2}{x} - \frac{3}{x^2} \right)^x$ .