Review for exam 3

Exercise 1. Evaluate the expressions

$$\log_8 18 - 2\log_8 3 + \log_8 16$$

$$e^{2\ln(5)} + 2^{2\log_2(3) - \log_2(5)}$$

Exercise 2. Solve

$$\ln(x+2) + \ln(x-1) = \ln(4)$$

$$\log_2(\log_3(\ln(x))) = 0$$

Exercise 3. Find the domain and the derivative of

$$f(x) = \cos(\ln(x)) + x^2 \ln(1 - 2x^2)$$

Exercise 4. Find the derivative of

$$f(x) = \ln\left(\frac{3x\sqrt{3 - x^3}\,e^x}{\sin^2 x}\right)$$

Exercise 5. A bacteria culture starts with 4000 bacteria and after 3 hours there are 12000 bacteria.
   Find an expression for the number of bacteria after t hours.
   Find the number of bacteria after 4 hours.
   When will the population reach 20000?

Exercise 6. After 3 days, a sample of radon-222 decayed to 58% of its original amount.
   What is the half-life of radon-222?
   How long would it take the sample to decay to 10% of its original amount.

Exercise 7. A bowl of boiling water is placed in a room where the temperature is 20°C. Its initial temperature is 100°C. 10 minutes later, the temperature of the bowl is 50°. find an expression for the temperature of the object t minutes later.

Exercise 8. Find the exact value of

$$\text{Arcsin}\left(\sin\left(\frac{4\pi}{3}\right)\right)$$

$$tan\,\text{Arccos}(0.3)$$

Exercise 9. Simplify

$$\cos(\text{Arctan}\,x).$$

Exercise 10. Find the derivative of

$$f(x) = \text{Arcsin}(2x - 1) + (\text{Arctan}\,x)^{-1} + \frac{\text{Arccos}\,x}{x}$$

Exercise 11. Find the limits

$$\lim_{x \to 0} \frac{e^{4x} - 1 - 4x}{x^2}$$

$$\lim_{x \to \infty} x^2 e^{-x}$$

$$\lim_{x \to 0} x^5 \ln(x)$$

$$\lim_{x \to 0} (\cos x)^{3/x}$$

$$\lim_{x \to 0} (\cos 3x)^{2 \cot x}$$

$$\lim_{x \to \infty} \ln \left(\frac{2x - 3}{2x + 5} \right)^{5x - 1}.$$

Exercise 12. Find the critical numbers of the functions $f(x) = x^{1/3}(x - 1)^{2/3}$.

Exercise 13. Find the absolute extrema of

1. $f(x) = x - 2 \cos x$ on $[-\pi, \pi]$

2. $f(x) = x\, e^x$ on $[0, 2]$.

Exercise 14. Let $f(x) = (x^2 - 4)^3$.

1. Find the intervals on which f is increasing or decreasing.

2. Find the local maximum and minimum values of f.

3. Find the intervals of concavity and the inflection points.

Exercise 15. If 1200cm^2 of material is available to make a box with square base and an open top, find the largest possible volume of the box.

Exercise 16. Find the point on the parabola $y = x^2$ that is closest to the point $(-3, 0)$.

Exercise 17. Find the most general antiderivative of

1. $f(x) = \sin x - 2\sqrt{x}$.

2. $f(x) = \sec^2 x + \dfrac{x^2 + 3x - 5}{x}$.

Exercise 18. Find $f(x)$ for

1. $f(x) = 1 + \dfrac{1}{x^2}, \quad x > 0, \quad f(1) = 3$.

2. $f'(x) = 4 - 3(1 + x^2)^{-1}, \quad f(1) = 0$.

Exercise 19. A ball is thrown at an angle of 45° to the ground. If the ball lands 90m away, what was its initial speed?