Section 6.1, 6.2, 6.3, 6.4

Exercise 1. Write the sum in sigma notation:

1. $\frac{3}{\sqrt{3}}+\frac{3}{\sqrt{4}}+\frac{3}{\sqrt{5}}+\frac{3}{\sqrt{6}}+\frac{3}{\sqrt{7}}+\frac{3}{\sqrt{8}}+\frac{3}{\sqrt{9}}+\frac{3}{\sqrt{10}}+\frac{3}{\sqrt{11}}$.
2. $n^2+(n+1)^2+(n+2)^2+(n+3)^2+(n+4)^2+(n+5)^2+(n+6)^2+(n+7)^2+(n+8)^2$.
3. $\frac{1}{2}-\frac{1}{4}+\frac{1}{6}-\frac{1}{8}+\frac{1}{10}-\frac{1}{12}+\frac{1}{14}-\frac{1}{16}+\frac{1}{18}-\frac{1}{20}$.

Exercise 2. Find the value of the sum

1. $\sum_{j=1}^{100} 2j+5$.
2. $\sum_{i=1}^{n}(3+2i)^2$.
3. $\sum_{k=1}^{43}(-1)^k$.
4. $\sum_{i=1}^{99}\left(\frac{1}{i}-\frac{1}{i+1}\right)$.
5. $\sum_{k=1}^{n}\left(i^5-(i-1)^5\right)$.

Exercise 3. Given the function $f(x)=1+\tan^2 x$ on the interval $[-\pi/4,\pi/4]$, the partition points $\{-\pi/4,-\pi/6,0\pi/6,\pi/4\}$ and x_i^*=left endpoint,

1. Sketch the graph of f and the approximating rectangles.
2. Find $||P||$.
3. Find the sum of the area of the approximating rectangles.

Exercise 4. Find the area under the curve $y=x^3$ from 0 to 1 using subintervals of equal length and taking x_i^* to be the left end points.

Same question with x_i^* being the right endpoint.

Exercise 5. Express each definite integral as a limit of a Riemann sum:

1. $\int_1^5 \frac{1}{1+x^2}\mathrm{d}x$.
2. $\int_{-1}^{1}\cos x\mathrm{d}x$.

Exercise 6. Express the limit as a definite integral

1. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\left(8\left(1+\frac{3i}{n}\right)^7 - 4\right)$.
2. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{5}{n}\left(\sqrt{1+\left(3+\frac{5i}{n}\right)^2}\right)$.

Exercise 7. Evaluate each integral by interpreting it in terms of areas

1. $\int_{-3}^{3} |x-2| - 1 \mathrm{d}x$
2. $\int_{1}^{4} \sqrt{9-(x-1)^2}\mathrm{d}x$

Exercise 8. Are the functions

$$f(x) = \begin{cases} 2x+5, & 0 \leqslant x \leqslant 1 \\ 3x-1, & 1 < x \leqslant 2 \end{cases}$$

and

$$g(x) = \begin{cases} (x-1)^{-1}, & x \neq 1 \\ 2 & x = 1 \end{cases}$$

integrable on the interval $[0,2]$?

Exercise 9. Evaluate the following integrals

1. $\int_{0}^{3} 3x^2 - 6x + 1 \mathrm{d}x$.
2. $\int_{0}^{\sqrt{3}} \frac{1}{1+x^2}\mathrm{d}x$.
3. $\int_{1}^{\pi} \sin x \mathrm{d}x$.
4. $\int_{\ln(5)}^{\ln(2)} \mathrm{e}^x \,\mathrm{d}x$.
5. $\int_{1}^{9} \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) \mathrm{d}x$.
6. $\int_{-5}^{3} 12x^2 \left|x-1\right| \mathrm{d}x$.
7. $\int_{\pi}^{\pi} \frac{\sin x}{x^2 + 2\sqrt{1+x^2}}\mathrm{d}x$.

Exercise 10. Find the derivative of

1. $\int_{1}^{x^2} \cos t^2 \mathrm{d}t$.
2. $\int_{\cos x}^{5} \frac{1}{t^4+1}\mathrm{d}t$.