
WIR 3
Sections 2.3, 2.5, 2.6

1 Section 2.3

1. Given that

$$\lim_{x \rightarrow 2^-} f(x) = 3, \quad \lim_{x \rightarrow 2^+} f(x) = 1, \quad \lim_{x \rightarrow 2} g(x) = 4.$$

(a) Find $\lim_{x \rightarrow 2^-} \frac{f(x)^2 - 3\sqrt{g(x)}}{f(x)g(x)}$.

(b) Find $\lim_{x \rightarrow 2^+} \frac{f(x)^2 - 3\sqrt{g(x)}}{f(x)g(x)}$ and $\lim_{x \rightarrow 2} \frac{f(x)^2 - 3\sqrt{g(x)}}{f(x)g(x)}$.

(c) Find $\lim_{x \rightarrow 2} \frac{f(x)^2 - 4f(x) + 1}{g(x)}$.

2. Evaluate each limit, if it exists:

(a) $\lim_{w \rightarrow 2} \sqrt[3]{\frac{5w - 2}{3w^2 + 2w - 1}}$.

(b) $\lim_{t \rightarrow -3} \frac{t^2 + t - 6}{t + 3}$.

(c) $\lim_{h \rightarrow 0} \frac{(5 + h)^2 - 25}{h}$.

(d) $\lim_{x \rightarrow 2} \frac{\frac{x + 3}{x - 1} - 5}{x - 2}$.

(e) $\lim_{x \rightarrow 4} \frac{4\sqrt{x - 3} - x}{x^2 - 16}$.

(f) $\lim_{t \rightarrow 1} \left\langle \frac{t^3 - 1}{t^2 - 1}, \sqrt{3t - 1} \right\rangle$.

(g) $\lim_{x \rightarrow 2} \frac{x^2 - 4}{|x - 2|}$.

(h) $\lim_{x \rightarrow 1} x^4 \cos\left(\frac{\pi}{x}\right)$.

(i) $\lim_{x \rightarrow 0} x^4 \cos\left(\frac{\pi}{x}\right)$.

3. Let $f(x)$ a function defined on $[0, 2]$ such that

$$x^2 + x + 1 \leq f(x) \leq x^3 + 2 \quad \text{for all } x \text{ in } [0, 2].$$

Can we find $\lim_{x \rightarrow 1} f(x)$?

4. Let f be a function defined by

$$f(x) = \begin{cases} 2x + 3 & \text{for } x < -2 \\ x^2 + 3x + 1 & \text{for } -2 \leq x \leq 1 \\ 3x - 2 & \text{for } 1 < x \end{cases}$$

Find

$$\begin{aligned} f(-2) = & \lim_{x \rightarrow -2^-} f(x) = & \lim_{x \rightarrow -2^+} f(x) = & \lim_{x \rightarrow -2} f(x) = \\ f(1) = & \lim_{x \rightarrow 1^-} f(x) = & \lim_{x \rightarrow 1^+} f(x) = & \lim_{x \rightarrow 1} f(x) = \end{aligned}$$

On which interval(s) is f continuous?

2 Section 2.5

1. Let f be the function defined by

$$f(x) = \begin{cases} \frac{\sqrt{x} - \sqrt{2}}{x - 2} & \text{for } x \neq 2 \\ \frac{\sqrt{2}}{4} & \text{for } x = 2 \end{cases}$$

Is f continuous on \mathbb{R} .

2. Let g be the function defined by

$$g(x) = \begin{cases} x^2 + 1 & \text{for } x < 2 \\ 3x - 1 & \text{for } 2 \leq x \leq 5 \\ 2x + 5 & \text{for } x > 5 \end{cases}$$

At which point is g discontinuous?

3. Determine a such that the function

$$g(x) = \begin{cases} x^2 + 3a & \text{for } x < 1 \\ ax - 2 & \text{for } 1 \leq x \end{cases}$$

is continuous on \mathbb{R} .

4. Determine whether the following functions f have a removable discontinuity at $x = a$? If the discontinuity a is removable, find a function g that agrees with f for all $x \neq a$ and is continuous at a ?

(a) $f(x) = \frac{(x-1)(x^2+5x+6)}{x+2}$.

(b) $f(x) = \frac{\sqrt{x}-2}{x^2-16}$.

(c) $f(x) = \frac{x^2-5x+4}{x+4}$.

5. Use the Intermediate Value Theorem to prove that the following equations have a solution on the interval $[0, 1]$.

(a) $x^4 - 3x^3 - 2x + 1 = 0$

(b) $\cos(\pi x) = x \sin(\pi x)$

3 Section 2.6

1. Calculate the following limits if they exist:

(a) $\lim_{x \rightarrow \infty} \frac{x^3 - 3x^4 + 2x + 5}{x^4 + x^2 - 1}$.

(b) $\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + 2x - 1}}{x - 5}$.

(c) $\lim_{x \rightarrow \infty} \sqrt{x + 6} - \sqrt{x - 4}$.

2. Find the vertical and horizontal asymptote of the following functions

(a) $f(x) = \frac{2x - 1}{3x - 4}$.

(b) $g(x) = \frac{(x^2 - 1)(x - 5)}{x^2 - 4x - 5}$.