WIR 3
Sections 2.3, 2.5, 2.6

1 Section 2.3

1. Given that
$$\lim_{x\to 2^-} f(x) = 3, \quad \lim_{x\to 2^+} f(x) = 1, \quad \lim_{x\to 2} g(x) = 4.$$

   (a) Find $\displaystyle\lim_{x\to 2^-} \frac{f(x)^2 - 3\sqrt{g(x)}}{f(x)g(x)}$.

   (b) Find $\displaystyle\lim_{x\to 2^+} \frac{f(x)^2 - 3\sqrt{g(x)}}{f(x)g(x)}$ and $\displaystyle\lim_{x\to 2} \frac{f(x)^2 - 3\sqrt{g(x)}}{f(x)g(x)}$.

   (c) Find $\displaystyle\lim_{x\to 2} \frac{f(x)^2 - 4f(x) + 1}{g(x)}$.

2. Evaluate each limit, if it exists:

(a) $\lim_{w \to 2} \sqrt[3]{\dfrac{5w-2}{3w^2+2w-1}}$.

(b) $\lim_{t \to -3} \dfrac{t^2+t-6}{t+3}$.

(c) $\lim_{h \to 0} \dfrac{(5+h)^2-25}{h}$.

(d) $\lim_{x \to 2} \dfrac{\dfrac{x+3}{x-1}-5}{x-2}$.

(e) $\lim_{x \to 4} \dfrac{4\sqrt{x-3}-x}{x^2-16}$.

(f) $\lim_{t \to 1} \langle \dfrac{t^3-1}{t^2-1}, \sqrt{3t-1} \rangle$.

(g) $\lim_{x \to 2} \frac{x^2 - 4}{|x - 2|}$.

(h) $\lim_{x \to 1} x^4 \cos\left(\frac{\pi}{x}\right)$.

(i) $\lim_{x \to 0} x^4 \cos\left(\frac{\pi}{x}\right)$.

3. Let $f(x)$ a function defined on $[0, 2]$ such that

$$x^2 + x + 1 \leqslant f(x) \leqslant x^3 + 2 \quad \text{for all } x \text{ in } [0, 2].$$

Can we find $\lim_{x \to 1} f(x)$?

4. Let f be a function defined by

$$f(x) = \begin{cases} 2x + 3 & \text{for } x < -2 \\ x^2 + 3x + 1 & \text{for } -2 \leqslant x \leqslant 1 \\ 3x - 2 & \text{for } 1 < x \end{cases}$$

Find

$f(-2) =$	$\lim_{x \to -2^-} f(x) =$	$\lim_{x \to -2^+} f(x) =$	$\lim_{x \to -2} f(x) =$
$f(1) =$	$\lim_{x \to 1^-} f(x) =$	$\lim_{x \to 1^+} f(x) =$	$\lim_{x \to 1} f(x) =$

On which interval(s) is f continuous?

2 Section 2.5

1. Let f be the function defined by

$$f(x) = \begin{cases} \dfrac{\sqrt{x}-\sqrt{2}}{x-2} & \text{for } x \neq 2 \\ \dfrac{\sqrt{2}}{4} & \text{for } x = 2 \end{cases}$$

   Is f continuous on $\mathbb{R}$.

2. Let g be the function defined by

$$g(x) = \begin{cases} x^2+1 & \text{for } x < 2 \\ 3x-1 & \text{for } 2 \leqslant x \leqslant 5 \\ 2x+5 & \text{for } x > 5 \end{cases}$$

   At which point(s) is g discontinuous?

3. Determine a such that the function

$$g(x) = \begin{cases} x^2+3a & \text{for } x < 1 \\ ax-2 & \text{for } 1 \leqslant x \end{cases}$$

   is continuous on $\mathbb{R}$.

4. Determine whether the following functions f have a removable discontinuity at $x = a$? If the discontinuity a is removable, find a function g that agrees with f for all $x \neq a$ and is continuous at a?

   (a) $f(x) = \dfrac{(x-1)(x^2+5x+6)}{x+2}$.

   (b) $f(x) = \dfrac{\sqrt{x}-2}{x^2-16}$.

   (c) $f(x) = \dfrac{x^2-5x+4}{x+4}$.

5. Use the Intermediate Value Theorem to prove that the following equations have a solution on the interval $[0, 1]$.

   (a) $x^4 - 3x^3 - 2x + 1 = 0$

   (b) $\cos(\pi x) = x \sin(\pi x)$

3 Section 2.6

1. Calculate the following limits if they exist:

   (a) $\lim_{x \to \infty} \frac{x^3 - 3x^4 + 2x + 5}{x^4 + x^2 - 1}$.

   (b) $\lim_{x \to -\infty} \frac{\sqrt{x^2 + 2x - 1}}{x - 5}$.

   (c) $\lim_{x \to \infty} \sqrt{x + 6} - \sqrt{x - 4}$.

2. Find the vertical and horizontal asymptote of the following functions

   (a) $f(x) = \frac{2x - 1}{3x - 4}$.

   (b) $g(x) = \frac{(x^2 - 1)(x - 5)}{x^2 - 4x - 5}$.