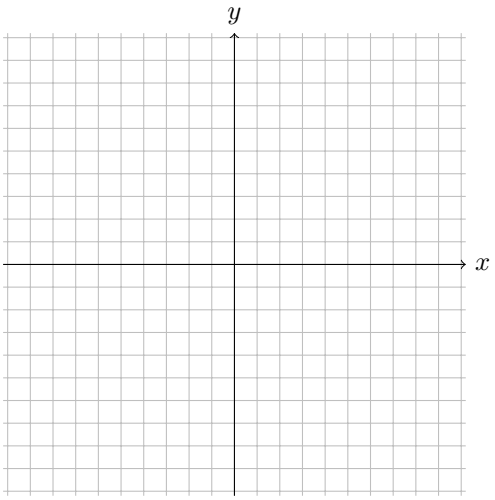
Review for Exam 1

1 Chapter 1

Exercise 1.

1. Draw the vectors $\overrightarrow{a} = \langle 2, 3\rangle$, $\overrightarrow{b} = \langle -1, 2\rangle$, and $\overrightarrow{c} = \langle 7, 0\rangle$.



2. Find the magnitude of $\overrightarrow{a}$.

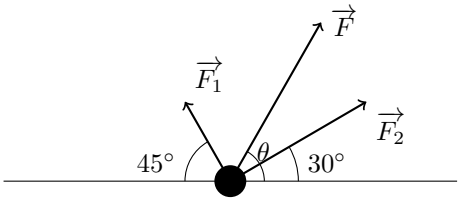
3. Find a unit vector in the opposite direction as $\overrightarrow{b}$.

4. Find a unit vector orthogonal to $\overrightarrow{b}$.

5. Find s and t such that $\overrightarrow{c} = s\overrightarrow{a} + t\overrightarrow{b}$.

Exercise 2. Two forces $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ with magnitude 6N and 8N acts on an object at a point P as shown infigure below.

Find the resulting force $\overrightarrow{F}$ acting at P as well as its magnitude and its direction (indicate the direction by finding the angle θ, shown in the picture.



Exercise 3. Find the value(s) of x such that $\langle x, x+1\rangle$ and $\langle x, 4\rangle$ are orthogonal.

Exercise 4. Find the scalar projection and the vector projection of $\overrightarrow{b} = \langle 3, 3\rangle$ onto $\overrightarrow{a} = \langle 2, -1\rangle$.

Exercise 5. Find the work done by a force of 20lb acting in the direction N30°W in moving an object 6ft due west.

Exercise 6. Given the parametric curve

$$x(t) = \frac{1-t}{1+t}, \quad y(t) = t^2.$$

Find a Cartesian equation of the curve.

Exercise 7. Find a vector equation, a parametric equation a Cartesian equation of the line passing through the point $(2, 3)$ and parallel to $\overrightarrow{v} = \langle -1, 4\rangle$.

Exercise 8. Find a vector equation, a parametric equation, a Cartesian equation of the line passing through $A(1, 3)$ and $B(4, -2)$.

Exercise 9. Let L_1 and L_2 be the lines determined by the vector equations

$$L_1 : \overrightarrow{r}_1(t) = \langle 8 + 3t, 10 + 4t \rangle$$
$$L_2 : \overrightarrow{r}_2(t) = \langle 3 + 8t, -5 - 6t \rangle$$

1. Does the line L_1 passes through the point $(14, 18)$? Does the line L_2 passes through the point $(14, 18)$?

2. Are the line orthogonal, parallel or neither?

3. In case the lines are not parallel, find the point of intersection

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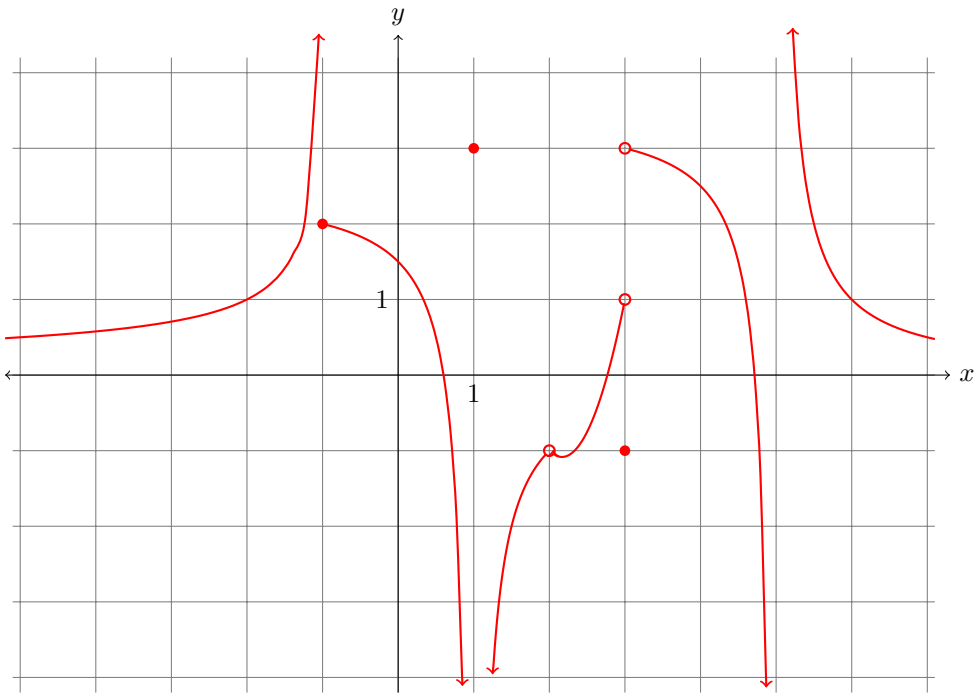
2 Chapter 2

Exercise 10.



Find the values of the function, the one side limits and the limits at $x = -1$, $x = 1$, $x = 2$, and $x = 5$.

Exercise 11. Find and classify all the points of discontinuity for the function

$$f(x) = \begin{cases} 2x+1 & \text{for } x \leqslant 1 \\ \sqrt{3x^2+x+5} & \text{for } 1 < x \leqslant 2 \\ 4x+5 & \text{for } 2 < x \end{cases}$$

Exercise 12. Find the limit of $\overrightarrow{r}(t) = (3t^2-1)\mathbf{i} + \left(\dfrac{t^2-1}{t+1}\right)\mathbf{j}$ at $t = 1$.

Exercise 13. Find the limit

$$\lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right) + 3$$

Exercise 14. Find the vertical and horizontal asymptotes of

$$f(x) = \frac{(x-3)(x^2-1)}{x^3-2x^2-3x}.$$

Exercise 15. Determine whether the following function have horizontal asymptote. If they do, give an equation of the asymptote.

1. $f(x) = \dfrac{(3x^2 - +5x1)}{x^3-2x^2-3x}$,

2. $f(x) = \dfrac{x^2+5x^5+7x^2-3}{x^2-3x^3+2x^4+9x^5}$.

3. $f(x) = \dfrac{3x^3 + 6x - 8}{7x^2 + 3x - 2}$.

4. $f(x) = \sqrt{(x^2 + 10x - 3)} - \sqrt{(x^2 - 8x + 5)}$.

5. $f(x) = \sqrt{(x^2 + 3x - 8)} + x$.

Exercise 16. At which point is the function $f(x) = \dfrac{|x^2 - 5x + 4|}{x - 1}$ not continuous?

Exercise 17. Does the function $f(x) = \dfrac{\sqrt{7 - 3x} - 1}{\sqrt{6 - x} - 2}$ have a removable discontinuity at $x = 2$?

Exercise 18. Determine the value of a such that the function

$$f(x) = \begin{cases} x^2 + ax - 3 & \text{for } x < 2 \\ 5x + a & \text{for } x \geqslant 2 \end{cases}$$

is continuous at $x = 2$.

3 Section 2.7-3.1

Exercise 19. Use the limit definition of the derivative to find the derivative

1. $f(x) = x^3 - 5x^2 + 2$ at $x = a$.

2. $f(x) = \dfrac{2x+1}{x-5}$ at $x = 2$.

Find an equation of the tangent line to the graph of f at $x = 2$.

3. $f(x) = \dfrac{7}{\sqrt{2x-1}}$ at $x = 5$.

Exercise 20. At which point(s) is the function $\left|x^2 - 9\right|$ not differentiable.

Exercise 21. Using the definition of the derivative, find the tangent vector to the curve

$$\overrightarrow{r}(t) = \langle t^2 + t, \sqrt{3t} \rangle$$

at $(4, 3)$.