
Sections 3.4-3.7

Exercise 1. Find the following limits:

1. $\lim_{x \rightarrow 0} \frac{\tan 5x}{\sin 3x}$.
2. $\lim_{x \rightarrow 0} \frac{\tan 3x(\cos x - 1)}{\sin^2 2x}$.
3. $\lim_{x \rightarrow 0} x \csc^2(3x) \tan x$.

Exercise 2. Find the derivative of

1. $f(x) = \frac{x^2}{\sin x - \cos x}$.
2. $f(x) = x^3 \tan x \sec x$.

Exercise 3. Find the derivatives of

1. $f_1(x) = \cos(\sqrt{x+5})$
2. $f_2(x) = \sqrt{\tan(3x)}$.
3. $f_3(x) = \left(\sqrt{x^3 - 4x^5 + 5x^7}\right) (3x^2 - 5x + 4)^5$.
4. $f_4(x) = \frac{\tan^2(3x)}{\sqrt{5x+2}}$.

Exercise 4. Find an equation of the tangent line to the curve $y = \sin(\sin(2x))$ at the point $(\pi, 0)$.

Exercise 5. Find an equation of the tangent line to the curve

$$y = \frac{25}{(7x-2)^3}$$

at the point $(1, 1/5)$.

Exercise 6. Find $\frac{dy}{dx}$ by implicit differentiation

1. $\cos(x-y) = x^2 - y^2$.
2. $2xy + 5y^3 = 3x^4$.

Exercise 7. Show that the given curves $x^2 + y^2 = 2x$ and $x^2 + y^2 = -2y$ are orthogonal.

Exercise 8. Given the implicit relation

$$4x^2 + y^4 = 4y^2$$

1. Find $\frac{dy}{dx}$.

2. Find an equation of the tangent line at the point $\left(\frac{\sqrt{3}}{2}, 1\right)$.
3. At what point(s) does the curve have a horizontal tangent line?

Exercise 9. Suppose the f and g are two differentiable function such that

$$f(0) = -4, \quad f'(0) = 3, \quad f(1) = 2, \quad f'(1) = -3, \quad g(1) = 0, \quad g'(1) = 7.$$

Find the derivative of $f(g(x^2))$ at $x = 1$.

Exercise 10. Sketch the curve defined by $\vec{r}(t) = \langle t^3 - 1, 3t \rangle$.

Exercise 11. Find the domain and the derivative of $\vec{r}(t) = \left\langle \frac{t^2}{2t-1}, \sqrt{3t-1} \right\rangle$.

Exercise 12. Find a tangent vector with unit length to the curve

$$\vec{r}(t) = \langle t \sin 2t, t \cos 2t \rangle$$

at the point with the parameter $t = \frac{3\pi}{8}$

Exercise 13. At what point do the curves traced by $\vec{r}_1(t) = \langle 3-t, 7-4t+t \rangle$ and $\vec{r}_2(s) = \langle s-2, s^2 \rangle$ intersect?

Exercise 14. Given the "fish curve" determined by

$$\vec{r}(t) = \langle \cos 2t, \cos 3t \rangle \quad t \in [0, \pi].$$

1. For which value(s) of t does the curve passes through the point $(.5, 1)$?
2. Find the angle between the 2 tangent lines at the point $(.5, 1)$.