Texas A&M University

Sections 3.2-4.2

Exercise 1. Find the limits

1. $\lim_{x\to\infty} \frac{e^{3x} - e^{2x}}{e^{3x} + 4e^{3x+5}}$.
2. $\lim_{x\to\pi^+} e^{1/\sin x}$.
3. $\lim_{x\to\infty} \frac{e^{3x}}{3e^{4x} + 2e^{2x}}$.
4. $\lim_{x\to-\infty} \frac{e^{3x}}{3e^{4x} + 2e^{2x}}$.

Exercise 2. Find the derivative of the following functions:

1. $f(x) = \frac{(\tan x)e^{-3x^2}}{x^2+1}$.
2. $f(x) = x^3 \cot(e^{2x^2-5})$.
3. $g(x) = e^{x \sec x}$.

Exercise 3. Find an equation of the tangent line to the curve $y = x^2 e^{\tan x}$ at $x = \pi/4$.

Exercise 4. Find an equation of the tangent line to the curve

$$e^{x^3y-4x} + 2 = x^2 - y^2$$

at the point $(2, 1)$.

Exercise 5. Determine whether the following functions are one-to-one. In case f is one-to-one, determine the inverse of f.

1. $f(x) = \sqrt{x^3 + 2x - 1}$.
2. $f(x) = \frac{x^3+2}{x^3-1}$.
3. $f(x) = x^3 - 3x^2$.
4. $f(x) = x^2 - 4x + 7$ for $x \geqslant 2$.

Exercise 6. Let g be the inverse of $f(x) = e^{3x} + 4x - 2$. Evaluate $g'(-1)$.

Exercise 7. Let f be a one-to-one function and g its inverse. Let $G(x) = x^3 e^{3g(x)}$ such that $f(1) = 2$, $f'(x) = 3$, $f(2) = 3$, $f'(2) = 5$. Find $G'(2)$.

Exercise 8. Find a linear and a quadratic approximation of the function $f(x) = \sqrt[3]{x}$ at $x = 8$. Find an approximation of $\sqrt[3]{7.9}$.

Exercise 9. Use a quadratic approximation to evaluate an approximation of $\sec 46°$.

Exercise 10. Use a linearization to evaluate $\cos^{-1}(.51)$.

Exercise 11. A kite 100ft above the ground moves horizontally at a speed of 8ft/s. At what rate is the angle between the string and the horizontal decreasing when 200ft of string have been let out?

Exercise 12. A water trough is 10m long and has the shape of an isosceles trapezoid that is 30cm wide at the bottom and 80cm wide at the to and has height 50cm. If the trough is being filled with water at a rate of $0.2\text{m}^3/\text{mn}$, how fast is the water level rising when the water is 30cm deep.

Exercise 13. given the curve

$$x(t) = \frac{2t}{1+t^3}, \quad y(t) = \frac{2t^2}{1+t^3}$$

1. Find an equation of the tangent line at the point with parameter 1.
2. Find an equation of the tangent line at the point $\left(\frac{4}{9}, \frac{8}{9}\right)$.
3. Find the points on the curve were the tangent are horizontal or vertical.

Exercise 14. Find an equation of the tangent line to the curve $\overrightarrow{r}(t) = \sec t\mathbf{i} + \tan t\mathbf{j}$.

Exercise 15. Find the second derivative of $f(x) = \dfrac{x^2}{x+1}$.

Exercise 16. Find the second derivative of y when y is given implicitly by

$$ye^{x^2} = x^2 + y^2.$$

Exercise 17. The curves $\overrightarrow{r}(t) = \langle t, t^3\rangle$ and $\overrightarrow{r}(t) = \langle \sin 2t, \sin\sqrt{3}t\rangle$ intersects at the origin. Find their angle of intersection.

Exercise 18. Show the the curves $y = x^2$ and $x^2 + 2y^2 = 20$ are orthogonal.

Exercise 19. Find the derivatives of the following functions

1. $f(x) = \sqrt{\tan(\sin x)}$.
2. $g(x) = \left(\frac{1-3x}{x^2+1}\right)^5$.
3. $h(x) = \left(\sec 3x + \csc 2x + \mathrm{e}^{x^2}\right)^3$.

Exercise 20. Given that $f(2) = 1$ $f'(2) = -3$, $g(3) = 2$, $g'(3) = -5$, $f(3) = 4$, $f'(3) = 7$.

Exercise 21. Find the derivative of $H(x) = \dfrac{(x^2-5x)f(g(x))}{g(x) - f(x)}$ at $x = 3$.

Exercise 22. Find the following limits

1. $\displaystyle\lim_{x\to 0} \frac{\sin 5x \cos 3x}{\tan 2x}$.
2. $\displaystyle\lim_{x\to 0} \frac{\tan(3x)(\cos 2x - 1)}{\sin^2(5x)}$.

Exercise 23. Find the equation of both tangent line to the parabola $y = x^2 + x + 3$ that passes through the point $(2, 0)$.

Exercise 24. Find the equations of the tangent lines to the curve $y = \dfrac{x-1)}{x+1}$ that are parallel to the line $x - 2y = 0$.