1 Chapter 1

Exercise 1.

- Find an equation of the line passing through the points $A(3,1)$ and $B(7,-4)$.
- Find an equation of the line parallel to the line (AB) and passing throught $C(10,9)$.
- Find an equation of the line perpendicular to (AB) and passing through C.

Exercise 2. (36p71) A particle is moving in the xy-plane and its position (x,y) at time t is given by

$$x(t) = 3t + 1, \quad y(t) = t^2 - t.$$

1. Find the position at time $t = 3$.
2. At what time is the particle at the point $(16,20)$.
3. Does the particle pass through the point $(7,4)$.

Exercise 3. (22p70) Find the distance from the point $(-1,2)$ to the line $y = 4x + 3$.

Exercise 4. (34p 61) Find the scalar and the vector projection of $\overrightarrow{b} = \langle 3,-1\rangle$ onto $\overrightarrow{a} = \langle 2,3\rangle$.

Exercise 5. (30p71) Eliminate the parameter to find a cartesian equation for the curve $x(t) = 1 + \cos t$, $y(t) = 1 + \sin^2 t$.

2 Chapter 2

2.1 Limits at finite points

Exercise 6. Compute

$$\lim_{x\to 2} \frac{3x^2 - x - 10}{x^2 - 4}.$$

Exercise 7. Find the vertical asymptotes of

$$f(x) = \frac{x^4 - 81}{2x^2 - 5x - 3}.$$

Exercise 8. Compute

$$\lim_{x\to -2} \frac{\frac{1}{x}+\frac{1}{2}}{x^3+8}.$$

Exercise 9. Compute

$$\lim_{x\to 4} \frac{3-\sqrt{x+5}}{x-4}.$$

Exercise 10. Compute

$$\lim_{x\to 27} \frac{x-27}{x^{1/3}-3}.$$

Exercise 11. Compute

$$\lim_{x\to 1} \frac{x^{1/3}-1}{x^{1/4}-1}.$$

Exercise 12. Compute

$$\lim_{x\to 0} \frac{\sin 5x}{3x}.$$

Exercise 13. Compute

$$\lim_{x\to 0} \frac{\cos(2x)-1}{\cos x-1}.$$

Exercise 14. Compute

$$\lim_{x\to 0} \frac{x^3-7x}{x^3}.$$

Exercise 15. Compute

$$\lim_{x\to 0} \frac{x^4+5x-3}{2-\sqrt{x^2+4}}.$$

Exercise 16. Compute

$$\lim_{x\to 1} \frac{x^3-1}{(x-1)^2}.$$

Exercise 17. Compute

$$\lim_{x\to \frac{\pi}{2}} \frac{\tan(2x)}{x-\frac{\pi}{2}}.$$

2.2 Continuity

Exercise 18. Consider the function

$$f(x)=\begin{cases} \dfrac{1}{x^2} & \text{if } x<-1 \\ 2x & \text{if } -1\leqslant x<1 \\ 3 & \text{if } x=1 \\ x+1 & \text{if } 1<x\leqslant 2 \\ \dfrac{-1}{(x-2)^2} & \text{if } x>2. \end{cases}$$

Determine the following limits.

1. (a) $\lim_{x\to -1^-} f(x)$.
   (b) $\lim_{x\to -1^+} f(x)$.
   (c) $\lim_{x\to -1} f(x)$.
   (d) $f(-1)$.
   (e) Is f continuous at -1?
2. (a) $\lim_{x\to 1^-} f(x)$.
   (b) $\lim_{x\to 1^+} f(x)$.
   (c) $\lim_{x\to 1} f(x)$.
   (d) $f(1)$.
   (e) Is f continuous at 1?
3. (a) $\lim_{x\to 2^-} f(x)$.
   (b) $\lim_{x\to 2^+} f(x)$.
   (c) $\lim_{x\to 2} f(x)$.
   (d) $f(2)$.
   (e) Is f continuous at 2?

Exercise 19. Consider the function

$$f(x) = \begin{cases} a+bx & \text{if } x < 2 \\ 3 & \text{if } x = 2 \\ b - ax^2 & \text{if } x > 2 \end{cases}$$

Determine the values of the constant a and b so that f is continuous at $x = 2$.

Exercise 20. Determine if the following function is continuous at $x = -2$

$$f(x) = \begin{cases} x^2 + 2x & \text{if } x \leqslant -2 \\ x^3 - 6x & \text{if } x > -2 \end{cases}$$

Exercise 21. Determine if the following function is continuous at $x = 0$

$$f(x) = \begin{cases} \dfrac{x-6}{x-3} & \text{if } x < 0 \\ 3x+2 & \text{if } x = 0 \\ \sqrt{4+x^2} & \text{if } x > 0 \end{cases}$$

Exercise 22. Determine if the function $h(x) = \dfrac{x^2-1}{x^3+1}$ is continuous at $x = -1$.

Exercise 23. Check the following function for continuity at $x = 3$ and $x = -3$.

$$f(x) = \begin{cases} \dfrac{x^3-27}{x^2-9} & \text{if } x \neq 3 \\ \frac{9}{2} & \text{if } x = 3 \end{cases}$$

Exercise 24. Determine all the values of the constant A and B so that the following function is continuous for all values of x.

$$f(x) = \begin{cases} Ax - B & \text{if } x \leqslant -1 \\ 2x^2 + 3Ax + B & \text{if } -1 < x < 1 \\ 5 & \text{if } x \geqslant 1 \end{cases}$$

For $A = 1$ and $B = 0$, is f differentiable at $x = 1$, and at $x = -1$

Exercise 25. Let

$$f(x) = \begin{cases} x^2 \cos\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

1. Show that f is continuous for all values of x.
2. Show that f is differentiable for all values of x.
3. Show that the derivative f' is not continuous at $x = 0$.

2.3 Limits at infinity

Exercise 26. Compute

$$\lim_{x \to -\infty} \frac{x+7}{3x+5}$$

.

Exercise 27. Compute

$$\lim_{x \to \infty} \frac{7x^2 + x - 100}{2x^2 - 5x}.$$

Exercise 28. Compute

$$\lim_{x \to \infty} \frac{7x^2 - x + 11}{4 - x}.$$

Exercise 29. Compute

$$\lim_{x \to \infty} \frac{x^2 - 3x + 7}{x^3 + 10x - 4}.$$

Exercise 30. Compute

$$\lim_{x \to \infty} \sqrt{\frac{x^3 + 7x}{4x^3 + 5}}.$$

Exercise 31. Compute

$$\lim_{x \to -\infty} x + \sqrt{x^2 + 7}.$$

Exercise 32. Compute

$$\lim_{x \to -\infty} x - \sqrt{x^2 + 7}.$$

Exercise 33. Find the horizontal asymptotes of

$$f(x) = \frac{x-3}{\sqrt{9x^2-5x}}.$$

Exercise 34. Compute

$$\lim_{x\to+\infty} \ln\left(\frac{x^6-100}{x^6+100}\right).$$

2.4 Intermediate Value theorem

Exercise 35. (50p147) Use the Intermediate Value Theorem to show that there is a solution of the equation

$$x^4 + 1 = \frac{1}{x}$$

in the interval $(0.5, 1)$.

3 Chapter 3

3.1 Derivatives

Exercise 36. Use the definition of the derivative to find $f'(3)$ for $fx) = x^2 - 2x$.

Exercise 37. Same question for $f(x) = \frac{1}{x}$.

Exercise 38. Find the derivative of

$$x\sin x + \sqrt[5]{3x^2+5} - \frac{2x+3}{7x-5}.$$

Exercise 39. (50p 235) Find the equation of the tangent line to the curve $y = x\sqrt{1+x^2}$ at the point $(1, \sqrt{2})$.

Exercise 40. Suppose that $f(2) = 3$, $g(2) = 5$, $f'(2) = -2$, $g'(2) = 4$, $f'(5) = 11$.
Find the derivative at $x = 2$ of the following functions

- $f(x)g(x)$
- $f(g(x))$
- $x^2 f(x) + (f^2(x)) - g(3x-4)$

3.2 Parametric curves

Exercise 41. (84p236) Find an equation of the tangent line to the curve

$$x(t) = t^6 + t^3, \quad y(t) = t^4 + t^2$$

at $t = 1$.

3.3 Implicit differentiation

Exercise 42. A curve is given implicitly by the equation $3xy+y^3 = 2x^2$. Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ at $(-1,-1)$.

Exercise 43. Find an equation of the tangent line to the cuve $\sqrt{x}+\sqrt{y} = 3$ at the point $(1,4)$.

Exercise 44. Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ at the point $(1,1)$ for the curve

$$\mathrm{e}^{xy-x} = \cos(y-x).$$

3.4 Higher derivatives

Exercise 45. A particle is moving forward along a straight track. After t seconds, the position is

$$s(t) = 60t - 32\ln(1+t) \text{ feet.}$$

What is the acceleration after 3 seconds.

Exercise 46. (71p238) A particle moves on a vertical line so that its coordinates at time t is $y = t^3 - 12t + 3$, $T \geqslant 0$.

1. Find the velocity and acceleration functions.
2. When is the particle moving upward and when is the particle moving downward?
3. Find the distance that the particle travels in the time interval $0 \leqslant t \leqslant 3$.

3.5 Linear, quadratic approximation

Exercise 47. Use a linear and a quadratic approximation to $f(x) = \sin(x)$ near $\pi/3$ to get an approximation of $\sin 1$.

3.6 Newton's method

Exercise 48. Starting with $x = 0$, find the 2 first approximations to the solution of $x^3 - x - 1 = 0$.

3.7 Problem

Exercise 49. (87p236) The volume of a cube is increasing at a rate of $10\text{cm}^3/\text{min}$. How fast is the surface area increasing when the length of an edge is 30cm.

Exercise 50. (88p236) A paper cup has the shape of a cone with height 10cm and radius 3cm (at the top). If the water is poured into the cup at the rate of $2\text{cm}^3/\text{s}$. how fast is the water level rising when the water is 5cm deep?

4 Chapter 4

4.1 Exponential functions

Exercise 51. Find the derivative of $\mathrm{e}^{\cos x}$.

Exercise 52. Find $\lim_{x\to 1^+} \mathrm{e}^{2/(x-1)}$.

Exercise 53. (50p249) For what values of λ does the function $y = \mathrm{e}^{\lambda x}$ satify the equation $y + y' = y''$.

Exercise 54. Find the limit $\lim_{x\to-\infty} \mathrm{e}^{x/3}\cos x$.

4.2 Inverse function

Exercise 55. Find the inverse of the function $f(x) = x^2 - 6x + 3$ on the interval $[3, +\infty)$.

Exercise 56. Let $f(x) = 3 + x + \mathrm{e}^x$ and g the inverse function of f. Find $g'(4)$.

4.3 Logarithm

Exercise 57. Find the derivative of $f(x) = \ln(x^6) - (\ln x)^6$.

Exercise 58. Solve $\ln(x^2 + 1) - 2\ln(x - 1) = 0$.

Exercise 59. Find the asymptotes to the curve $y = \dfrac{\ln x}{1 + \ln x}$

Exercise 60. Find the derivative of $f(x) = \dfrac{\mathrm{e}^x \sin^2 x(x-9)^3}{\sqrt[3]{7x-1}(2x+6)^5}$.

4.4 Exponential growth and decay

Exercise 61. (3p274) A bacteria culure starts with 500 bacteria and after 3 hours there are 8000 bacteria.

1. Find an expression for the number of bacteria after t hours.
2. When will the population reach 30,000?

Exercise 62. From exam 3.

4.5 Inverse trigonometric function

Exercise 63. Find the derivative of $\sin^{-1}(x^3) + \cos^{-1}(x^5)$.

Exercise 64. If $f(x) = x\tan^{-1} x$, find $f'(1)$.

Exercise 65. Find the derivative of $\sin^{-1} x + \cos^{-1} x$.
Conclude that for all $x \in [-1, 1]$, $\sin^{-1} x + \cos^{-1} x = \pi/2$.

4.6 L'Hospital's rule

exercices in other sections.

5 Chapter 5

5.1 Variation of f

Exercise 66. Let $f(x) = x^4 - 13x^2 + 36$.

- Determine the interval(s) where the function f is increasing, decreasing.
- Determine the local maxima, the local minima

Exercise 67. Suppose that P is a polynomial function such that $P(2) = -1$, $P'(2) = 3$, $P''(2) = 5$.

1. Which of the following statments are TRUE?
   (a) P is increasing at $x = 2$,
   (b) P is concave up at $x = 2$,
   (c) P is decreasing at $x = 2$,
   (d) P is concave down at $x = 2$,
   (e) P has an inflection point at $x = 2$
   (f) P has a critical point at $x = 2$
2. Find an equation of the tangent line at the point where $x = 2$.
3. Use a quadratic approximation to estimate $P(1.9)$.

Exercise 68. On which interval is the function $f(x) = 3x - x^3$ increasing?

Exercise 69. Find the absolute maxima and the absolute minima of the function $f(x) = x - 2\cos x$ on the interval $[-\pi, \pi]$.

5.2 Applied maximum and minimum problems

Exercise 70. Find the dimension of the rectangle with largest area that can be inscribed in a circle of radius r.

Exercise 71. Exercise from exam 3.

Exercise 72. (62p358) A metal storage tank with volume V is to be constructed in the shape of a right circular cylinder surmounted by a hemisphere. What dimensions will require the least amount of metal.

5.3 Antiderivatives

Exercise 73. Find antiderivatives for

1. $f(x) = x^2 + \sqrt[5]{x} = \frac{1}{x^2} - \frac{1}{\sqrt{x}} + \frac{1}{x}$.
2. $f(x) = 1 + 2\sin x + \cos x$ such that $F(0) = 3$.
3. $f(x) = \mathrm{e}^x - \dfrac{1}{x}$ such that $F(1) = 0$
4. $f(x) = \dfrac{1}{\sqrt{1 - x^2}}$.

Exercise 74. Find the position if the acceleration is given.

6 Chapter 6

6.1 Riemann sum

Exercise 75. The interval $[0, \pi/2]$ is devided into subintervals by the partition set

$$\left\{0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}\right\}.$$

If $f(x) = 4\cos x$ and x_i^* is chosen to be the left end point of the ith subinterval, find the Riemann sum.

Exercise 76. From exam 3.

6.2 Definite integrals

Exercise 77. Find

1. $\int_3^9 \frac{1}{x}\mathrm{d}x$
2. $\int_4^9 \sqrt{x} - \frac{1}{\sqrt{x}}\mathrm{d}x.$
3. $\int_{-3}^1 (x-1)(x+3)\mathrm{d}x.$
4. $\int_{\ln 2}^{\ln 5} \mathrm{e}^{2x}\,\mathrm{d}x$
5. $\int_{\sqrt{2}/2}^{1/2} \frac{1}{\sqrt{1-x^2}}\mathrm{d}x.$

6.3 Substitutions

Exercise 78. Evaluate

$$\int \frac{\sin x}{2+\cos x}\mathrm{d}x$$

Exercise 79. Find

1. $\int \frac{x^2}{\sqrt{1-x}}\mathrm{d}x.$
2. $\int \cos(5x-6)\mathrm{d}x.$
3. $\int \cos^7 t \sin t \mathrm{d}t.$
4. $\int_0^1 (x-1)\sqrt{x^2-2x+7}\mathrm{d}x$
5. ...see you book, exercises in class.