Review For final

1 Chapter 1

Exercise 1.

- Find an equation of the line passing through the points $A(3,1)$ and $B(7,-4)$.

  Answer. `They are infinitely many equations. Here are 2 of them.`

  `Parametric equation:`

  $$\boxed{x(t) = 3 + 4t, \quad y(t) = 1 - 5t}$$

  `Cartesian equation:`

  $$\boxed{y = \frac{-4}{5}x + \frac{17}{5}}$$

- Find an equation of the line parallel to the line (AB) and passing throught $C(10,9)$.

  Answer.

  `Parametric equation:`

  $$\boxed{x(t) = 10 + 4t, \quad y(t) = 9 - 5t}$$

  `Cartesian equation:`

  $$\boxed{y = \frac{-4}{5}x + 17}$$

- Find an equation of the line perpendicular to (AB) and passing through C.

  Answer.

  `Parametric equation:`

  $$\boxed{x(t) = 10 + 5t, \quad y(t) = 9 + 4t}$$

  `Cartesian equation:`

  $$\boxed{y = \frac{5}{4}x - \frac{7}{2}}$$

Exercise 2. (36p71) A particle is moving in the xy-plane and its position (x,y) at time t is given by

$$x(t) = 3t + 1, \quad y(t) = t^2 - t.$$

1. Find the position at time $t = 3$.

   Answer.

   $$\boxed{(10, 6)}$$

2. At what time is the particle at the point $(16, 20)$.

   Answer.

   $$\boxed{t = 5}$$

3. Does the particle pass through the point $(7, 4)$.

   Answer.

   $$\boxed{\text{No}}$$

Exercise 3. (22p70) Find the distance from the point $(-1, 2)$ to the line $y = 4x + 3$.

Answer.

$$\boxed{\frac{3}{\sqrt{17}}}$$

Exercise 4. (34p 61) Find the scalar and the vector projection of $\overrightarrow{b} = \langle 3, -1 \rangle$ onto $\overrightarrow{a} = \langle 2, 3 \rangle$.

Exercise 5. (30p71) Eliminate the parameter to find a cartesian equation for the curve $x(t) = 1 + \cos t$, $y(t) = 1 + \sin^2 t$.

Answer. `Since` $-1 \leqslant \cos t \leqslant 1$, $0 \leqslant x \leqslant 2$.

$$\boxed{(x-1)^2 + y = 2 \quad \text{for } 0 \leqslant x \leqslant 2}$$

2 Chapter 2

2.1 Limits at finite points

Exercise 6. Compute

$$\lim_{x \to 2} \frac{3x^2 - x - 10}{x^2 - 4}.$$

Answer.

$$\boxed{\frac{11}{4}}$$

Exercise 7. Find the vertical asymptotes of

$$f(x) = \frac{x^4 - 81}{2x^2 - 5x - 3}.$$

Answer.

$$\boxed{x = -\frac{1}{2}}$$

Exercise 8. Compute

$$\lim_{x\to -2} \frac{\frac{1}{x}+\frac{1}{2}}{x^3+8}.$$

Answer.

$$\boxed{x = -\frac{1}{48}}$$

Exercise 9. Compute

$$\lim_{x\to 4} \frac{3-\sqrt{x+5}}{x-4}.$$

Answer.

$$\boxed{\frac{-1}{6}}$$

Exercise 10. Compute

$$\lim_{x\to 27} \frac{x-27}{x^{1/3}-3}.$$

Answer.

$$\boxed{27}$$

Exercise 11. Compute

$$\lim_{x\to 1} \frac{x^{1/3}-1}{x^{1/4}-1}.$$

Answer.

$$\boxed{\frac{4}{3}}$$

Exercise 12. Compute

$$\lim_{x\to 0} \frac{\sin 5x}{3x}.$$

Answer.

$$\boxed{\frac{5}{3}}$$

Exercise 13. Compute

$$\lim_{x\to 0} \frac{\cos(2x)-1}{\cos x - 1}.$$

Answer.

$$\boxed{4}$$

Exercise 14. Compute

$$\lim_{x\to 0} \frac{x^3-7x}{x^3}.$$

Answer.

$$\boxed{-\infty}$$

Exercise 15. Compute

$$\lim_{x\to 0} \frac{x^4+5x-3}{2-\sqrt{x^2+4}}.$$

Answer.

$$\boxed{+\infty}$$

Exercise 16. Compute

$$\lim_{x\to 1}\frac{x^3-1}{(x-1)^2}.$$

Answer.

$$\boxed{\text{does not exist}}$$

Exercise 17. Compute

$$\lim_{x\to\frac{\pi}{2}}\frac{\tan(2x)}{x-\frac{\pi}{2}}.$$

Answer.

$$\boxed{2}$$

2.2 Continuity

Exercise 18. Consider the function

$$f(x)=\begin{cases}\dfrac{1}{x^2} & \text{if } x<-1\\ 2x & \text{if } -1\leqslant x<1\\ 3 & \text{if } x=1\\ x+1 & \text{if } 1<x\leqslant 2\\ \dfrac{-1}{(x-2)^2} & \text{if } x>2.\end{cases}$$

Determine the following limits.

1. (a) $\lim_{x\to -1^-} f(x)$.

   Answer.

   $$\boxed{1}$$

   (b) $\lim_{x\to -1^+} f(x)$.

   Answer.

   $$\boxed{-2}$$

   (c) $\lim_{x\to -1} f(x)$.

   Answer.

   $$\boxed{\text{Does not exist}}$$

   (d) $f(-1)$.

   Answer.

   $$\boxed{-2}$$

   (e) Is f continuous at -1?

   Answer.

   $$\boxed{\text{No}}$$

2. (a) $\lim_{x\to 1^-} f(x)$.

   Answer.

   $$\boxed{2}$$

   (b) $\lim_{x\to 1^+} f(x)$.

   Answer.

   $$\boxed{2}$$

   (c) $\lim_{x\to 1} f(x)$.

   Answer.

   $$\boxed{2}$$

   (d) $f(1)$.

   Answer.

   $$\boxed{3}$$

   (e) Is f continuous at 1?

   Answer.

   $$\boxed{\text{No}}$$

3. (a) $\lim_{x\to 2^-} f(x)$.

   Answer.

   $$\boxed{3}$$

   (b) $\lim_{x\to 2^+} f(x)$.

   Answer.

   $$\boxed{-\infty}$$

   (c) $\lim_{x\to 2} f(x)$.

   Answer.

   $$\boxed{\text{Does not exist}}$$

   (d) $f(2)$.

   Answer.

   $$\boxed{3}$$

   (e) Is f continuous at 2?

   Answer.

   $$\boxed{\text{No}}$$

Exercise 19. Consider the function

$$f(x) = \begin{cases} a + bx & \text{if } x < 2 \\ 3 & \text{if } x = 2 \\ b - ax^2 & \text{if } x > 2 \end{cases}$$

Determine the values of the constant a and b so that f is continuous at $x = 2$.
Answer.

$$\boxed{a = \frac{-1}{3}, b = \frac{5}{3}}$$

Exercise 20. Determine if the following function is continuous at $x = -2$

$$f(x) = \begin{cases} x^2 + 2x & \text{if } x \leqslant -2 \\ x^3 - 6x & \text{if } x > -2 \end{cases}$$

Answer. $\lim_{x \to -2} f(x)$ `does not exist`

$$\boxed{\text{The function is not continuous at } x = -2}$$

Exercise 21. Determine if the following function is continuous at $x = 0$

$$f(x) = \begin{cases} \dfrac{x-6}{x-3} & \text{if } x < 0 \\ 3x + 2 & \text{if } x = 0 \\ \sqrt{4 + x^2} & \text{if } x > 0 \end{cases}$$

Answer. $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0) = 2$

$$\boxed{\text{The function is continuous at } x = 0}$$

Exercise 22. Determine if the function $h(x) = \dfrac{x^2 - 1}{x^3 + 1}$ is continuous at $x = -1$.

Answer. `The function is not defined at` $x = -1$.

$$\boxed{\text{The function is not continuous at } x = -1.}$$

Exercise 23. Check the following function for continuity at $x = 3$ and $x = -3$.

$$f(x) = \begin{cases} \dfrac{x^3 - 27}{x^2 - 9} & \text{if } x \neq 3 \\ \frac{9}{2} & \text{if } x = 3 \end{cases}$$

Answer.

$$\boxed{\text{The function is continuous at } x = 3 \text{ and not continuous at } x = -3 \text{ (not defined at } x = -3)}$$

Exercise 24. Determine all the values of the constant A and B so that the following function is continuous for all values of x.

$$f(x) = \begin{cases} Ax - B & \text{if } x \leqslant -1 \\ 2x^2 + 3Ax + B & \text{if } -1 < x < 1 \\ 5 & \text{if } x \geqslant 1 \end{cases}$$

Answer.

$$\boxed{A = 1, B = 0}$$

For $A = 1$ and $B = 0$, is f differentiable at $x = 1$, and at $x = -1$?

Answer.

$$\boxed{f \text{ is differentiable at } x = -1 \text{ and is not differentiable at } x = 1.}$$

Exercise 25. Let

$$f(x) = \begin{cases} x^2 \cos\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

1. Show that f is continuous for all values of x.

   Answer. `Use the squeeze theorem`

$$\boxed{\lim_{x\to 0} f(x) = 0 = f(0)}$$

2. Show that f is differentiable for all values of x.

   Answer. `Differentiable away from 0 since` f `is a composite of differentiable functions. At` $x=0$`, no rules to find the derivative, so we use the definition of the derivative. Using the squeeze theorem,`

$$\boxed{f'(0) = \lim_{x\to 0} \frac{f(x)-f(0)}{x} = 0}$$

3. Show that the derivative f' is not continuous at $x = 0$.

   Answer. $\lim_{x\to 0} f'(x)$ `does not exist.`

$$\boxed{f' \text{ is not continuous at } x = 0.}$$

2.3 Limits at infinity

Exercise 26. Compute

$$\lim_{x\to -\infty} \frac{x+7}{3x+5}$$

. **Answer.**

$$\boxed{\frac{1}{3}}$$

Exercise 27. Compute

$$\lim_{x\to \infty} \frac{7x^2+x-100}{2x^2-5x}.$$

Answer.

$$\boxed{\frac{7}{2}}$$

Exercise 28. Compute

$$\lim_{x\to \infty} \frac{7x^2-x+11}{4-x}.$$

Answer.

$$\boxed{-\infty}$$

Exercise 29. Compute

$$\lim_{x\to \infty} \frac{x^2-3x+7}{x^3+10x-4}.$$

Answer.

$$\boxed{0}$$

Exercise 30. Compute

$$\lim_{x\to\infty}\sqrt{\frac{x^3+7x}{4x^3+5}}.$$

Answer.

$$\boxed{\frac{1}{2}}$$

Exercise 31. Compute

$$\lim_{x\to-\infty} x+\sqrt{x^2+7}.$$

Answer. `inderterminate, multiply and divide by the conjugate`

$$\boxed{0}$$

Exercise 32. Compute

$$\lim_{x\to-\infty} x-\sqrt{x^2+7}.$$

Answer. `No indeterminated form`

$$\boxed{-\infty}$$

Exercise 33. Find the horizontal asymptotes of

$$f(x)=\frac{x-3}{\sqrt{9x^2-5x}}.$$

Answer. $\lim_{x\to+\infty} f(x)=\frac{1}{3}$, $\lim_{x\to-\infty} f(x)=\frac{-1}{3}$

$$\boxed{y=\frac{1}{3},\quad y=\frac{-1}{3}}$$

Exercise 34. Compute

$$\lim_{x\to+\infty}\ln\left(\frac{x^6-100}{x^6+100}\right).$$

Answer.

$$\boxed{0}$$

2.4 Intermediate Value theorem

Exercise 35. (50p147) Use the Intermediate Value Theorem to show that there is a solution of the equation

$$x^4+1=\frac{1}{x}$$

in the interval $(0.5,1)$.

Answer. $f(x)=x^4+1-\frac{1}{x}$. f is continuous on the interval [0.5,1], $f(0.5)=\frac{-15}{16}<0$, $f(1)=1>0$. By the intermediate values theorem, there is a real $c\in(0.5,1)$ such that $f(c)=0$

$$\boxed{c^4+1=\frac{1}{c}}$$

3 Chapter 3

3.1 Derivatives

Exercise 36. Use the definition of the derivative to find $f'(3)$ for $fx) = x^2 - 2x$.
Answer. `The points for such a question will be given for the method more than for the result.`
$f'(3) = \lim_{h\to 0} \frac{f(3+h)-f(3)}{h} = \lim_{x\to 3} \frac{f(x)-f(3)}{x-3}$. `Find the limit without using L'Hospital's rule.`

$$\boxed{f'(3) = 4}$$

Exercise 37. Same question for $f(x) = \frac{1}{x}$.
Answer.

$$\boxed{f'(3) = \frac{-1}{9}}$$

Exercise 38. Find the derivative of

$$x \sin x + \sqrt[5]{3x^2+5} - \frac{2x+3}{7x-5}.$$

Answer. `Use the rules`

$$\boxed{f'(x) = \sin x + x\cos x + \frac{6x}{5}(3x^2+5)^{-4/5} + \frac{31}{(7x-5)^2}}$$

Exercise 39. (50p 235) Find the equation of the tangent line to the curve $y = x\sqrt{1+x^2}$ at the point $(1, \sqrt{2})$.
Answer. `The slope is` $y'(1)$.

$$\boxed{y = \frac{3}{\sqrt{2}}(x-1) + \sqrt{2}}$$

Exercise 40. Suppose that $f(2) = 3$, $g(2) = 5$, $f'(2) = -2$, $g'(2) = 4$, $f'(5) = 11$.

Find the derivative at $x = 2$ of the following functions

- $h(x) = f(x)g(x)$.

  Answer.

  $$\boxed{h'(2)' = 3 \times 4 + (-2) \times 5 = 2}$$

- $j(x) = f(g(x))$.

  Answer.

  $$\boxed{j'(2) = 4 \times 11 = 44}$$

- $k(x) = x^2 f(x) + (f^2(x)) - g(3x-4)$.

  Answer.

  $$\boxed{k'(2) = (4 \times 3 + 4 \times (-2)) + (2 \times (-2) \times 3) + 3 \times 4 = 4}$$

3.2 Parametric curves

Exercise 41. (84p236) Find an equation of the tangent line to the curve

$$x(t) = t^6 + t^3, \quad y(t) = t^4 + t^2$$

at $t = 1$.

Answer. `The slope is` $\dfrac{y'(1)}{x'(1)}$.

$$\boxed{y = \frac{2}{3}(x-2) + 2}$$

3.3 Implicit differentiation

Exercise 42. A curve is given implicitly by the equation $3xy + y^3 = 2x^2$. Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ at $(1,1)$.

Answer. $3y + 3xy' + 3y'y^2 = 4x$

$$\boxed{\frac{\mathrm{d}y}{\mathrm{d}x} = y' = \frac{1}{6}}$$

Exercise 43. Find an equation of the tangent line to the cuve $\sqrt{x} + \sqrt{y} = 3$ at the point $(1,4)$.

Answer. $\dfrac{1}{2\sqrt{x}} + \dfrac{y'}{2\sqrt{y}} = 0$

$$\boxed{\frac{\mathrm{d}y}{\mathrm{d}x} = -2}$$

Exercise 44. Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ at the point $(1,1)$ for the curve

$$\mathrm{e}^{xy-x} = \cos(y-x).$$

Answer. $(y + xy' - 1)\,\mathrm{e}^{xy-x} = -(y'-1)\sin(y-x)$

$$\boxed{\frac{\mathrm{d}y}{\mathrm{d}x} = 0}$$

3.4 Higher derivatives

Exercise 45. A particle is moving forward along a straight track. After t seconds, the position is

$$s(t) = 60t - 32\ln(1+t) \text{ feet}.$$

What is the acceleration after 3 seconds.

Answer. `Find` $s''(3)$

$$\boxed{a(3) = 2 \text{ feet/s}^2}$$

Exercise 46. (71p238) A particle moves on a vertical line so that its coordinates at time t is $y = t^3 - 12t + 3$, $t \geqslant 0$.

1. Find the velocity and acceleration functions.

   Answer.

   $$\boxed{v(t) = 3t^2 - 12, \quad a(t) = 6t}$$

2. When is the particle moving upward and when is the particle moving downward?

   Answer. `The particle is moving upward if` $v(t) > 0$. `The particle is moving downward if` $v(t) < 0$.

   $$\boxed{\textit{upward}: \quad t > 2, \qquad \textit{downward}: \quad 0 \leqslant t < 2}$$

3. Find the distance that the particle travels in the time interval $0 \leqslant t \leqslant 3$.

   Answer. `The total distance is the distance between` $t = 0$ `and` $t = 2$ `plus the distance between` $t = 2$ `and` $t = 3$.

   $$\boxed{d = |y(2) - y(0)| + |y(3) - y(2)| = 52 \text{ feet}}$$

3.5 Linear, quadratic approximation

Exercise 47. Use a linear and a quadratic approximation to $f(x) = \sin(x)$ near $\pi/3$ to get an approximation of $\sin 1$.

3.6 Newton's method

Exercise 48. Starting with $x_1 = 0$, find the 2 first approximations to the solution of $x^3 - x - 1 = 0$.
Answer. $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$$\boxed{x_2 = -1 \quad x_3 = \frac{-1}{2}}$$

3.7 Problem

Exercise 49. (87p236) The volume of a cube is increasing at a rate of $10\text{cm}^3/\text{min}$. How fast is the surface area increasing when the length of an edge is 30cm.
Answer. `Differentiate` $v = a^3$

$$\boxed{\frac{\mathrm{d}a}{\mathrm{d}t} = \frac{1}{27}}$$

Exercise 50. (88p236) A paper cup has the shape of a cone with height 10cm and radius 3cm (at the top). If the water is poured into the cup at the rate of $2\text{cm}^3/\text{s}$. How fast is the water level rising when the water is 5cm deep?

4 Chapter 4

4.1 Exponential functions

Exercise 51. Find the derivative of $\mathrm{e}^{\cos x}$.

Exercise 52. Find $\lim_{x\to 1^+} \mathrm{e}^{2/(x-1)}$.

Exercise 53. (50p249) For what values of λ does the function $y = \mathrm{e}^{\lambda x}$ satify the equation $y + y' = y''$.
Answer.

$$\boxed{\lambda = \frac{1 \pm \sqrt{5}}{2}}$$

Exercise 54. Find the limit $\lim_{x\to -\infty} \mathrm{e}^{x/3} \cos x$.

4.2 Inverse function

Exercise 55. Find the inverse of the function $f(x) = x^2 - 6x + 3$ on the interval $[3, +\infty)$.

Exercise 56. Let $f(x) = 3 + x + \mathrm{e}^x$ and g the inverse function of f. Find $g'(4)$.

4.3 Logarithm

Exercise 57. Find the derivative of $f(x) = \ln(x^6) - (\ln x)^6$.
Answer. $f(x) = 6\ln x - (\ln x)^6$.

$$\boxed{f'(x) = \frac{6}{x} - \frac{6}{x}(\ln x)^5}$$

Exercise 58. Solve $\ln(x^2 + 1) - 2\ln(x + 1) = 0$.
Answer.

$$\boxed{x = 0}$$

Exercise 59. Find the asymptotes to the curve $y = \dfrac{\ln x}{1 + \ln x}$

Answer. `No vertical asymptotes at` $x = 0$,
`1 vertical asymptoe at` $x = e^{-1}$,
`1 horizontal asymptote at` $+\infty$,
`not defined for` $x < 0$ `so no horizontal asymptote at` $-\infty$

$$\boxed{x = \mathrm{e}^{-1}, \quad y = 1}$$

Exercise 60. Find the derivative of $f(x) = \dfrac{\mathrm{e}^x \sin^2 x (x-9)^3}{\sqrt[3]{7x-1}(2x+6)^5}$.

4.4 Exponential growth and decay

Exercise 61. (3p274) A bacteria culure starts with 500 bacteria and after 3 hours there are 8000 bacteria.

1. Find an expression for the number of bacteria after t hours.

   Answer.

$$\boxed{y = 500\,\mathrm{e}^{\frac{\ln 16}{3}t}}$$

2. When will the population reach 30,000?

   Answer.

$$\boxed{t = \frac{3\ln 60}{\ln 16}}$$

Exercise 62. From exam 3.

4.5 Inverse trigonometric function

Exercise 63. Find the derivative of $\sin^{-1}(x^3) + \cos^{-1}(x^5)$.
Answer.

$$\boxed{f'(x) = \frac{3x^2}{\sqrt{1-x^6}} - \frac{5x^4}{\sqrt{1-x^{10}}}}$$

Exercise 64. If $f(x) = x\tan^{-1} x$, find $f'(1)$.

Answer.

$$\boxed{f'(x) = \tan^{-1}(1) + \frac{1}{1+1^2} = \frac{\pi}{4} + \frac{1}{2}}$$

Exercise 65. Find the derivative of $h(x) = \sin^{-1} x + \cos^{-1} x$.

Answer.

$$\boxed{h'(x) = 0}$$

Conclude that for all $x \in [-1, 1]$, $\sin^{-1} x + \cos^{-1} x = \pi/2$.
Answer. $h'(x) = 0$, there exists a real constant c such that $h(x) = c$.

$$h(0) = c = \sin^{-1} 0 + \cos^{-1} 0 = 0 + \frac{\pi}{2}.$$

$$\boxed{c = \frac{\pi}{2}}$$

4.6 L'Hospital's rule

exercices in other sections.

5 Chapter 5

5.1 Variation of f

Exercise 66. Let $f(x) = x^4 - 12x^2 + 36$.

- Determine the interval(s) where the function f is increasing, decreasing.

  Answer.

f increasing for	$-\sqrt{8} \leqslant x \leqslant 0$	or for	$x \geqslant \sqrt{8}$
f decreasing for	$x \leqslant -\sqrt{8}$	or for	$0 \leqslant x \leqslant \sqrt{8}$

- Determine the local maxima, the local minima

  Answer.

Local maximum at	$x = 0$
Local minima at	$x = \pm\sqrt{8}$

Exercise 67. Suppose that P is a polynomial function such that $P(2) = -1$, $P'(2) = 3$, $P''(2) = 5$.

1. Which of the following statments are TRUE?

   (a) P is increasing at $x = 2$,

   (b) P is concave up at $x = 2$,

   (c) P is decreasing at $x = 2$,

   (d) P is concave down at $x = 2$,

   (e) P has an inflection point at $x = 2$

   (f) P has a critical point at $x = 2$

   Answer.

   $$\boxed{\#1, \#2}$$

2. Find an equation of the tangent line at the point where $x = 2$.

   Answer.

   $$\boxed{y = 3(x-2) - 1}$$

3. Use a quadratic approximation to estimate $P(1.9)$.

   Answer. $y \approx P(2) + P'(2)(x-2) + P''(2)\frac{(x-2)^2}{2}$

   $$\boxed{P(1.9) \approx -1.275}$$

Exercise 68. On which interval is the function $f(x) = 3x - x^3$ increasing?
Answer.

$$\boxed{[-1, 1]}$$

Exercise 69. Find the absolute maxima and the absolute minima of the function $f(x) = x - 2\cos x$ on the interval $[-\pi, \pi]$.
Answer. `critical points:` $-\pi/6$ `and` $-5\pi/6$

$$\boxed{hh}$$

5.2 Applied maximum and minimum problems

Exercise 70. Find the dimension of the rectangle with largest area that can be inscribed in a circle of radius r.
Answer. `Check that the critical point of the area function is a maximum`

$$\boxed{\sqrt{2}r}$$

Exercise 71. Exercise from exam 3.

Exercise 72. (62p358) A metal storage tank with volume V is to be constructed in the shape of a right circular cylinder surmounted by a hemisphere. What dimensions will require the least amount of metal.

5.3 Antiderivatives

Exercise 73. Find antiderivatives for

1. $f(x) = x^2 + \sqrt[5]{x} = \frac{1}{x^2} - \frac{1}{\sqrt{x}} + \frac{1}{x}$.
2. $f(x) = 1 + 2\sin x + \cos x$ such that $F(0) = 3$.
3. $f(x) = \mathrm{e}^x - \dfrac{1}{x}$ such that $F(1) = 0$
4. $f(x) = \dfrac{1}{\sqrt{1 - x^2}}$.

Exercise 74. Find the position if the acceleration is given.

6 Chapter 6

6.1 Riemann sum

Exercise 75. The interval $[0, \pi/2]$ is devided into subintervals by the partition set

$$\left\{0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}\right\}.$$

If $f(x) = 4\cos x$ and x_i^* is chosen to be the left end point of the ith subinterval, find the Riemann sum.

Exercise 76. From exam 3.

6.2 Definite integrals

Exercise 77. Find

1. $\int_3^9 \frac{1}{x} \mathrm{d}x$
2. $\int_4^9 \sqrt{x} - \frac{1}{\sqrt{x}} \mathrm{d}x.$
3. $\int_{-3}^1 (x-1)(x+3) \mathrm{d}x.$
4. $\int_{\ln 2}^{\ln 5} \mathrm{e}^{2x}\,\mathrm{d}x$
5. $\int_{\sqrt{2}/2}^{1/2} \frac{1}{\sqrt{1-x^2}} \mathrm{d}x.$

6.3 Substitutions

Exercise 78. Evaluate

$$\int \frac{\sin x}{2+\cos x} \mathrm{d}x$$

Exercise 79. Find

1. $\int \frac{x^2}{\sqrt{1-x}} \mathrm{d}x.$
2. $\int \cos(5x-6) \mathrm{d}x.$
3. $\int \cos^7 t \sin t \mathrm{d}t.$
4. $\int_0^1 (x-1)\sqrt{x^2-2x+7} \mathrm{d}x$
5. ...see you book, exercises in class.