Review for the final exam

Exercise 1. Let $A(1.3)$, $B(4,5)$, $C(3,2)$, and $D(8,4)$.

1. Find a unit vector orthogonal to $\overrightarrow{AC}$.
2. Find s such that $\overrightarrow{AB} + s\overrightarrow{AC}$ is orthogonal to $\overrightarrow{BC}$.
3. Find s and t such that $\overrightarrow{AD} = s\overrightarrow{AC} + t\overrightarrow{AB}$.

Exercise 2. Ropes 3m and 5m in length are fastened to a flag that is suspended inside an indoor stadium. The flag weights 10 pounds. The ropes, fastened at different heights, makes an angle of 45° and 60° with the horizontal.

Find the tension in each wire and the magnitude of each tension.

Exercise 3. (56p61) A wagon is pulled along a straight line from its initial position $(2,-1)$ to its final position $(5,9)$ by a constant horizontal force of 50N. How much work is done?

Exercise 4. (33p68) Determine whether the lines

$$L_1 : \overrightarrow{r}_1(t) = (2-t)\mathbf{i} + (-3+5t)\mathbf{j}$$

$$L_2 : \overrightarrow{r}_2(t) = (8+10t)\mathbf{i} + (2+2t)\mathbf{j}$$

are parallel, perpendicular, or neither. If the lines are not parallel, find their point of intersection.

Exercise 5. (36p69) An object is moving in the xy-plane and its position after t seconds is

$$\overrightarrow{r}(t) = \langle t-3, t^2-2t\rangle.$$

1. Find the position of the object at time $t = 5$.
2. Find the tangent vector at $t = 5$.
3. At what time is the object at the point $(1,8)$?
4. Does the object pass through the point $(3,20)$?
5. Find a Cartesian equation of the trajectory.

Exercise 6. (8p 90)

1. Sketch the graph of the function

$$g(x) = \begin{cases} 2-x & \text{if } x < -1 \\ x & \text{if } -1 \leqslant x < 1 \\ 4 & \text{if } x = 1 \\ 4-x & \text{if } x > 1 \end{cases}$$

2. Use the graph from part 1. to state the value of each of the following limits, if it exists.

$$\lim_{x\to 1^-} g(x) = \qquad \lim_{x\to 1^+} g(x) = \qquad \lim_{x\to 1} g(x) =$$

$$\lim_{x\to -1^-} g(x) = \qquad \lim_{x\to -1^-} g(x) = \qquad \lim_{x\to -1^-} g(x) =$$

3. Is the function g continuous at $x=-1$? at $x=1$?

Exercise 7. Determine the infinite limit

$$\lim_{x\to 2}\frac{5}{x^2-3x+2}= \qquad \lim_{x\to 2}\frac{3}{(x-2)^2}=$$

Exercise 8. Find the following limits or say why they don't exist.

1. $\lim_{x\to 5}\frac{|x-5|}{x-5}$.
2. $\lim_{x\to 3}\langle\frac{x^3-27}{x^2-9},\frac{\sqrt{4x+4}-\sqrt{5x+1}}{x^2-9}\rangle$.
3. $\lim_{x\to -\infty}\sqrt{x^2+3x+1}-\sqrt{x^2-4x+3}$.

Exercise 9. Find the horizontal and vertical asymptotes of

1. $f(x)=\frac{\sqrt{x^2-1}}{2x-2}$.
2. $g(x)=\frac{\sqrt{2x+3}-\sqrt{3x}}{x^2-3x}$.
3. $m(x)=\frac{3x^2-3x+6}{x^2+3x+2}$.

Exercise 10. Let

$$g(x)=\begin{cases}2x-x^2 & \text{if } 0\leqslant x\leqslant 2\\ 2-x & \text{if } 2<x\leqslant 3\\ x-4 & \text{if } 3<x<4\\ \pi & \text{if } x\geqslant 4\end{cases}$$

1. For each of the numbers 2, 3, and 4, determine whether g is continuous at that point.
2. For each of the numbers 2, 3, and 4, determine whether g is differentiable at that point.

Exercise 11. (39p121) Find the values of c and d that make h continuous on $\mathbb{R}$.

$$h(x)=\begin{cases}2x & \text{if } x<1\\ cx^2+d & \text{if } 1\leqslant x\leqslant 2\\ 4x & \text{if } x>2\end{cases}$$

Exercise 12. Find the derivative of $f(x)=\frac{2-x}{2+x}$ using the definition of derivative.

Exercise 13. (49p168) How many tangent lines to the curve $y=\frac{x}{(x+1)}$ pass through the point $(1,2)$? At which point do these tangent lines touch the curve.

Exercise 14. Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ for

$$\cos(xy)-2\tan\left(\frac{x}{y}\right)=1$$

Exercise 15. (35p199) Show that the curves

$$2x^2 + y^2 = 3, \quad x = y^2.$$

are orthogonal.

Exercise 16. Find an equation of the tangent to the curve $\overrightarrow{r}(t) = \langle t\sin t, t\cos t\rangle$ at the point corresponding to the parameter $t = \pi$.

Exercise 17. Find a linearization $L(x)$ of the function $f(x) = \dfrac{2}{\operatorname{Arctan} x}$ at $x = 1$

Find a quadratic approximation of $f(x) = \sec x$ at $x = 0$.

Exercise 18. Show the function $f(x) = \dfrac{1+3x}{1-2x}$ is one-to-one.

Exercise 19. Find the inverse of

$$f(x) = \ln\left(\frac{1 + 2e^{x^3}}{5 - 7\,\mathrm{e}^{x^3}}\right).$$

Exercise 20. Let g be the inverse of the function $f(x) = x^3 + 5x + \mathrm{e}^x$.

Find the derivative of $g(\cos(2x))$ at $x = 0$.

Exercise 21. Find the domain and solve the equations

1. $\ln(2\,\mathrm{e}^x - 1) = 3$
2. $\ln(x) + \ln(x-1) = \ln(.5)$
3. $\log_2(\log_3(\log_4(x))) = 0$

Exercise 22. Find the following limits

1. $\displaystyle\lim_{x\to 0} \frac{\operatorname{Arctan} x - \operatorname{Arcsin} x}{\tan x - \sin x}$.
2. $\displaystyle\lim_{x\to\infty} \mathrm{e}^{-x}\, x \ln x$.
3. $\displaystyle\lim_{x\to 0^+} \operatorname{Arctan}(\ln 3x)$.
4. $\displaystyle\lim_{x\to\infty} \left(\frac{3+x}{x-1}\right)^{5x}$.
5. $\displaystyle\lim_{x\to\infty} (\mathrm{e}^x + x)^{1/x}$.

Exercise 23. Find the exact value of $\cos \operatorname{Arcsin} 0.7$.

Exercise 24. $f(x) = x^5(x-1)^6$

1. Find the intervals on which f is increasing or decreasing?
2. Find the local minimum, local maximum values of f.
3. Find the intervals of concavity and the inflection points.

Exercise 25. Find the derivative of

1. $g(x) = \ln\left(\dfrac{x^2 \sin x}{(\operatorname{Arctan} x)\sqrt{x^2+3}}\right)$

2. $f(x) = x^{\ln(x)} - (\ln x)^x$.

Exercise 26. Using the logarithmic differentiation, find the derivative of

$$f(x) = \sqrt{\frac{(\cos^6 x)(e^{3x^2})\sqrt{x^5 - 1}}{(2x+1)x^{2/5}}}$$

Exercise 27. Find y' if $y = \ln(x^2 + y^2)$.

Exercise 28. Evaluate the definite integral of

1. $\displaystyle\int_{-0.5}^{0.5} \frac{1}{\sqrt{1-x^2}} \mathrm{d}x$
2. $\displaystyle\int_{1}^{3} 4x^3 - 6x^2 + 6x - 2\mathrm{d}x$

Exercise 29. Given the function $f(x) = 10 - x^2$ on the interval $[1, 3]$ using 4 subintervals of equal length and taking x_i^* to be the right endpoint.

1. Sketch the graph of f and the approximating rectangles.
2. Evaluate the area of the approximating rectangles.
3. Using rectangles of equal length and x_i^* being the right endpoints of the ith subinterval. Find the area between the x axis, $x = 1$, $x = 3$, the graph of f.

Exercise 30. Evaluate the sum

$$\sum_{i=3}^{9} 9(i-3)(i+1)$$