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Review for exam 1

1 Appendix D

Exercise 1. Given that $\sin\theta = \frac{3}{7}$ and $\frac{\pi}{2} \leqslant \theta \leqslant \pi$, Find the other trigonometric ratios.

Exercise 2. Find all the values of x in $[0, 2\pi]$ that satisfy the equation

$$4\cos x + \sin 2x = .0$$

Exercise 3. Find all the values of x in $[0, 2\pi]$ that satisfy the equation

$$2\cos 2x \cos x = 1 + \sin 2x \sin x.$$

2 Sections 1.1-1.3

Exercise 4. Let $A(1.3)$, $B(4, 5)$, $C(6, 1)$, and $D(-2, 2)$.

1. Find the magnitude of $\overrightarrow{AB} + 4\overrightarrow{BC}$.
2. Find a unit vector orthogonal to $\overrightarrow{AC}$.
3. Find the orthogonal vector projection of $\overrightarrow{AB}$ onto $\overrightarrow{AC}$.
4. Find s such that $\overrightarrow{AB} + s\overrightarrow{AC}$ is orthogonal to $\overrightarrow{BC}$.
5. Find s and t such that $\overrightarrow{AD} = s\overrightarrow{AC} + t\overrightarrow{AB}$.

Exercise 5. Ropes 3m and 5m in length are fastened to a flag that is suspended inside an indoor stadium. The flag weights 10 pounds. The ropes, fastened at different heights, makes an angle of 45° and 60° with the horizontal.

Find the tension in each wire and the magnitude of each tension.

Exercise 6. (32p61) For what values c is the angle between the vectors $\langle 1, 1\rangle$ and $\langle 1, c\rangle$ equal to 60°.

Exercise 7. (21,25 p 61) Determine whether the given vectors are parallel, orthogonal or neither.

1. $\overrightarrow{a} = \langle 2, -4\rangle$ and $\overrightarrow{b} = \langle -1, 2\rangle$.
2. $\overrightarrow{c} = 3\mathbf{i} + \mathbf{j}$ and $\overrightarrow{d} = -3\mathbf{i} + \mathbf{j}$.

Exercise 8. (56p61) A wagon is pulled a distance of 100m along a horizontal path by a constant force of 50N. The handle of the wagon is at an angle of 30° above the horizontal. How much work is done?

Exercise 9. (47p61) Find the distance between the parallel lines $y = 2x + 3$ and $y - 2x = 9$.

Exercise 10. Find a Cartesian equation for the curves

1. $x = 2 + \cos t$, $y = 3 - \sin t$.

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2. $x = 2t - 1$, $y = 2t^2 + 6t - 1$.

Exercise 11. Find a Cartesian equation and a vector equation of the line L_1 passing through $A(1, 3)$ and $B(4, 1)$.

Exercise 12. Find a Cartesian equation and a vector equation of the line passing through $A(1, 3)$ and parallel to the vector $\langle 2, -3\rangle$.

Exercise 13. (33p68) Determine whether the lines

$$L_1 : \overrightarrow{r}_1(t) = (2 - t)\mathbf{i} + (-3 + 5t)\mathbf{j}$$

$$L_2 : \overrightarrow{r}_2(t) = (8 + 10t)\mathbf{i} + (2 + 2t)\mathbf{j}$$

are parallel, perpendicular, or neither. If the lines are not parallel, find their point of intersection.

Exercise 14. (36p69) An object is moving in the xy-plane and its position after t seconds is

$$\overrightarrow{r}(t) = \langle t - 3, t^2 - 2t\rangle.$$

1. Find the position of the object at time $t = 5$.
2. Find the tangent vector at $t = 5$.
3. At what time is the object at the point $(1, 8)$?
4. Does the object pass through the point $(3, 20)$?
5. Find an equation in x and y whose graph is the path of the object.

3 Limits

Exercise 15. (8p 90)

1. Sketch the graph of the function

$$g(x) = \begin{cases} 2 - x & \text{if } x < -1 \\ x & \text{if } -1 \leqslant x < 1 \\ 4 & \text{if } x = 1 \\ 4 - x & \text{if } x > 1 \end{cases}$$

2. Use the graph from part 1. to state the value of each of the following limits, if it exists.

$$\lim_{x\to 1^-} g(x) = \qquad \lim_{x\to 1^+} g(x) = \qquad \lim_{x\to 1} g(x) =$$
$$\lim_{x\to -1^-} g(x) = \qquad \lim_{x\to -1^-} g(x) = \qquad \lim_{x\to -1^-} g(x) =$$

3. Is the function g continuous at $x = -1$? at $x = 1$?

Exercise 16. Determine the infinite limit

$$\lim_{x\to 5^-} \frac{6}{x-5} = \qquad \lim_{x\to 5^+} \frac{6}{x-5} =$$
$$\lim_{x\to 5} \frac{6}{x-5} = \qquad \lim_{x\to 2} \frac{3}{(x-2)^2} =$$

Exercise 17. Find the following limits or say why they don't exist.

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1. $\lim_{x\to 5} \dfrac{|x-5|}{x-5}$.
2. $\lim_{x\to\infty} \dfrac{x^3+4x^2-6x+7}{x^2-3x+5}$.
3. $\lim_{x\to 3} \langle \dfrac{x^3-27}{x^2-9}, \dfrac{\sqrt{4x+4}-\sqrt{5x+1}}{x^2-9} \rangle$.
4. $\lim_{x\to-\infty} \sqrt{x^2+3x+1}-\sqrt{x^2-4x+3}$.

Exercise 18. Find the horizontal and vertical asymptotes of

1. $f(x) = \dfrac{\sqrt{x^2-9}}{2x-6}$.
2. $g(x) = \dfrac{\sqrt{2x+3}-\sqrt{3x}}{x^2-3x}$.
3. $m(x) = \dfrac{3x^2-3x+6}{x^2+3x+2}$.

4 Continuity

Exercise 19. Let

$$g(x) = \begin{cases} 2x-x^2 & \text{if } 0 \leqslant x \leqslant 2 \\ 2-x & \text{if } 2 < x \leqslant 3 \\ x-4 & \text{if } 3 < x < 4 \\ \pi & \text{if } x \geqslant 4 \end{cases}$$

1. For each of the numbers 2, 3, and 4, determine whether g is continuous at that point.
2. For each of the numbers 2, 3, and 4, determine whether g is differentiable at that point.

5 Derivatives

Exercise 20. (54p147) Let $\overrightarrow{r}(t) = \langle \sqrt{x+1}, 2t-3t^2 \rangle$.

1. Find an tangent vector to the curve given by the graph of $\overrightarrow{r}(t)$ at the point where $t=3$.
2. Find a vector equation for the tangent line to the curve at the point where $t=3$.
3. Find a Cartesian equation for the tangent line to the curve at the point where $t=3$.

Exercise 21. Find the derivative of the following functions using the definition of derivative.

1. $g(x) = x^3 - x^2 + 3x$
2. $v(x) = \dfrac{4-3x}{2+x}$.
3. $u(x) = \dfrac{1}{\sqrt{x-1}}$.

Exercise 22. Determine where the function $\left|x^2-5x+6\right|$ is not differentiable, why?