Texas A&M University

Review for exam 1

1 Appendix D

Exercise 1. Answer.

$$\boxed{\cos\theta = -\frac{2\sqrt{10}}{7}, \quad \tan\theta = \frac{3}{2\sqrt{10}} = \frac{3\sqrt{10}}{20}, \quad \sec\theta = -\frac{7}{2\sqrt{10}} = -\frac{7\sqrt{10}}{20}, \quad \csc\theta = \frac{7}{3}, \quad \cot\theta = \frac{2\sqrt{10}}{3}}$$

Exercise 2. Answer.

$$4\cos x + 2\sin x\cos x = 0$$

$$2\cos x(2+\sin x) = 0$$

$\sin x \in [-1,1]$ `therefore` $\sin x + 2 \in [1,3]$ `is never zero.`

$$\cos x = 0$$

$$\boxed{x = \frac{\pi}{2}, \quad x = \frac{3\pi}{2}}$$

Exercise 3.

$$2\cos 2x\cos x = 1 + 2\sin 2x\sin x.$$

Answer.

$$2\cos(2x+x) = 1$$

`If` $x \in [0, 2\pi]$`, then` $3x \in [0, 6\pi]$ `and`

$$\cos 3x = \frac{1}{2}$$

`In` $[0, 6\pi]$`,` $3x = \frac{\pi}{3}, \quad \frac{5\pi}{3}, \quad \frac{7\pi}{3}, \quad \frac{11\pi}{3}, \quad \frac{13\pi}{3}, \quad \frac{17\pi}{3}.$

$$\boxed{x = \frac{\pi}{9}, \quad \frac{5\pi}{9}, \quad \frac{7\pi}{9}, \quad \frac{11\pi}{9}, \quad \frac{13\pi}{9}, \quad \frac{17\pi}{9}}$$

2 Sections 1.1-1.3

Exercise 4. Let $A(1.3)$, $B(4,5)$, $C(6,1)$, and $D(-2,2)$.

1. **Answer.**

$$\overrightarrow{AB} = \langle 3, 2\rangle, \quad \overrightarrow{BC} = \langle 2, -4\rangle.$$

$$\overrightarrow{AB} + 4\overrightarrow{BC} = \langle 3+8, 2+(-16)\rangle = \langle 11, -14\rangle$$

$$\left|\left|\overrightarrow{AB} + 4\overrightarrow{BC}\right|\right| = \sqrt{11^2 + 14^2}$$

$$\boxed{\left|\left|\overrightarrow{AB} + 4\overrightarrow{AC}\right|\right| = \sqrt{317}}$$

2. **Answer.** $\overrightarrow{AC} = \langle 5, -2\rangle$. Its orthogonal complement $\overrightarrow{v} = \overrightarrow{AC}^{\perp} = \langle 2, 5\rangle$ is orthogonal to $\overrightarrow{AC}$ but is not a unit vector.
$\frac{1}{||\overrightarrow{v}||}\overrightarrow{v}$ is a unit vector with the same direction as $\overrightarrow{v}$.
$||\overrightarrow{v}|| = \sqrt{2^2+5^2} = \sqrt{29}$.

$$\boxed{\langle \frac{2}{\sqrt{29}}, \frac{5}{\sqrt{29}}\rangle = \langle \frac{2\sqrt{29}}{29}, \frac{5\sqrt{29}}{29}\rangle. \quad \text{Second solution:} \langle \frac{-2}{\sqrt{29}}, \frac{-5}{\sqrt{29}}\rangle = \langle \frac{-2\sqrt{29}}{29}, \frac{-5\sqrt{29}}{29}\rangle.}$$

3. **Answer.** Use the formula from your textbook:

$$proj_{\overrightarrow{AC}}\overrightarrow{AB} = \frac{\overrightarrow{AB}\cdot\overrightarrow{AC}}{\left|\left|\overrightarrow{AC}\right|\right|^2}\overrightarrow{AC}.$$

$$proj_{\overrightarrow{AC}}\overrightarrow{AB} = \frac{15-4}{29}\langle 5, -2\rangle$$

$$\boxed{proj_{\overrightarrow{AC}}\overrightarrow{AB} = \langle \frac{55}{29}, -\frac{22}{29}\rangle}$$

4. **Answer.**

$$\overrightarrow{AB} + s\overrightarrow{AC} = \langle 3+5s, 2-2s\rangle$$

$\overrightarrow{AB} + s\overrightarrow{AC}$ is orthogonal to $\overrightarrow{BC}$ if their dot product is 0.

$$\langle 3+5s, 2-2s\rangle \cdot \langle 2, -4\rangle = 2(3+5s) - 4(2-2s) = 0$$

$$6 + 10s - 8 + 8s = 0$$

$$\boxed{s = \frac{2}{18} = \frac{1}{9}}$$

5. **Answer.**

$$\overrightarrow{AD} = \langle -3, -1\rangle, \qquad s\overrightarrow{AC} + t\overrightarrow{AD} = \langle 5s+3t, -2s+2t\rangle$$

$$\begin{cases} -3 = 5s+3t \\ -1 = -2s+2t \end{cases} \qquad \begin{cases} t = s - \frac{1}{2} \\ 5s + 3\left(s - \frac{1}{2}\right) = -3 \end{cases}$$

$$\boxed{s = \frac{-3}{16}, \quad t = -\frac{11}{16}}$$

Exercise 5. Answer. Let T_1 and T_2 be the tension of $\overrightarrow{T1}$ and $\overrightarrow{T_2}$ respectively and $\overrightarrow{W}$ the weight.

$$\overrightarrow{T_1} = \langle -T_1\cos 45, T_1\sin 45\rangle, \quad \overrightarrow{T_2} = \langle T_2\cos 60, T_2\sin 60\rangle, \quad \overrightarrow{W} = \langle 0, -10\rangle.$$

$$\overrightarrow{T_1} = \langle -T_1\frac{\sqrt{2}}{2}, T_1\frac{\sqrt{2}}{2}\rangle, \quad \overrightarrow{T_2} = \langle \frac{T_2}{2}, \frac{T_2\sqrt{3}}{2}\rangle, \quad \overrightarrow{W} = \langle 0, -10\rangle.$$

The Flag is in equilibrium, therefore

$$\overrightarrow{T_1} + \overrightarrow{T_2} + \overrightarrow{W} = 0$$

$$\langle -\frac{T_1\sqrt{2}}{2} + \frac{T_2}{2}, \frac{T_1\sqrt{2}}{2} + \frac{T_2\sqrt{3}}{2} - 10\rangle = \langle 0, 0\rangle$$

$$\begin{cases} -\frac{T_1\sqrt{2}}{2} + \frac{T_2}{2} = 0 \\ \frac{T_1\sqrt{2}}{2} + \frac{T_2\sqrt{3}}{2} - 10 = 0 \end{cases} \qquad \begin{cases} T_2 = T_1\sqrt{2} \\ T_1\sqrt{2} + T_1\sqrt{2}\sqrt{3} = 20 \end{cases}$$

$$T_1 = \frac{10\sqrt{2}}{1+\sqrt{3}}, \quad T_2 = 10(\sqrt{3}-1).$$

$$\boxed{T_1 = \frac{10\sqrt{2}}{1+\sqrt{3}}, \quad T_2 = 10(\sqrt{3}-1), \quad \overrightarrow{T_1} = \langle -\frac{10}{1+\sqrt{3}}, \frac{10}{1+\sqrt{3}} \rangle, \quad \overrightarrow{T_2} = \langle 5(\sqrt{3}-1), 5(3-\sqrt{3}) \rangle.}$$

Exercise 6. (32p61) **Answer.** Let $\overrightarrow{A}$ be the vector $\langle 1,1 \rangle$ and $\overrightarrow{C}$ be the vector $\langle 1,c \rangle$.
The angle between $\overrightarrow{A}$ and $\overrightarrow{C}$ is $60°$ if $\overrightarrow{A} \cdot \overrightarrow{C} = \left|\left|\overrightarrow{A}\right|\right| \left|\left|\overrightarrow{C}\right|\right| \cos 60°$.

$$1 + c = \sqrt{2}\sqrt{1+c^2}\left(\frac{1}{2}\right)$$

$$(1+c)^2 = \frac{1+c^2}{2} \quad \text{and} \quad 1 + c \geqslant 0$$

$$2(1+2c+c^2) = 1 + c^2 \quad \text{and} \quad 1 + c \geqslant 0$$

$$1 + 4c + c^2 = 0 \quad \text{and} \quad 1 + c \geqslant 0$$

$$\boxed{c = \sqrt{3} - 2}$$

Exercise 7. (21,25 p 61) **Answer.** See your book

□

Exercise 8. (56p61) A wagon is pulled a distance of 100m along a horizontal path by a constant force of 50N. The handle of the wagon is at an angle of 30° above the horizontal. How much work is done?
Answer. $W = \overrightarrow{F} \cdot \overrightarrow{D}$

$$W = 50 * 100 * \cos(30°)$$

$$\boxed{W = 2500\sqrt{3}J \approx 4330J}$$

Exercise 9. (47p61)**Answer.** Pick on point in the second line for example $P(0,9)$ and find the distance between P and the first line using the usual method (see the steps in http://such-that.com/TAMU/Distance-point-droite.php) See your textbook for the answers.

□

Exercise 10. Find a Cartesian equation for the curves

1. **Answer.**

$$\cos^2 t + \sin^2 t = 1$$

with

$$\cos t = x - 2, \quad \sin t = 3 - y$$

$$\boxed{(x-2)^2 + (3-y)^2 = 1}$$

2. **Answer.**

$$t = \frac{x+1}{2}, \quad y = 2\left(\frac{x+1}{2}\right)^2 + 6\left(\frac{x+1}{2}\right) - 1$$

$$\boxed{y = \frac{x^2}{2} + 4x + \frac{5}{2}}$$

Exercise 11. Answer. `Vector equation:` $\overrightarrow{AB} = \langle 3, -2 \rangle$,

$$\overrightarrow{r}(t) = \langle 1, 3 \rangle + t\langle 3, -2 \rangle = \langle 1+3t, 3-2t \rangle$$

`Cartesian equation: The slope is given by` $\dfrac{y_b - y_a}{x_b - x_a} = \dfrac{-2}{3}$.
`Using the slope-point equation:`

$$y - y_a = \frac{-2}{3}(x - x_a)$$

$$\boxed{\overrightarrow{r}(t) = \langle 1, 3 \rangle + t\langle 3, -2 \rangle = \langle 1+3t, 3-2t \rangle, \quad y = \frac{-2}{3}(x-1) + 3}$$

Exercise 12. Answer. `Vector equation:`

$$\overrightarrow{r}(t) = \langle 1, 3 \rangle + t\langle 2, -3 \rangle = \langle 1+2t, 3-3t \rangle$$

`Cartesian equation: The slope is given by` $m = \dfrac{-3}{2}$.
`Using the slope point equation` $y - 3 = \frac{-3}{2}(x-1)$

$$\boxed{\overrightarrow{r}(t) = \langle 1, 3 \rangle + t\langle 2, -3 \rangle = \langle 1+2t, 3-3t \rangle \qquad y = \frac{-3}{2}(x-1) + 3}$$

Exercise 13. (33p68) **Answer.** `See your textbook. The line are orthogonal.`
`To find the intersection point, solve` $\overrightarrow{r_1}(t) = \overrightarrow{r_2}(s)$.

$$(2-t)\mathbf{i} + (-3+5t)\mathbf{j} = (8+10s)\mathbf{i} + (2+2s)\mathbf{j}$$

$$\begin{cases} 2 - t = 8 + 10s \\ -3 + 5t = 2 + 2s \end{cases} \qquad \begin{cases} t = -6 - 10s \\ -3 + 5(-6 - 10s) = 2 + 2s \end{cases} \qquad \begin{cases} t = \dfrac{19}{26} \\ s = \dfrac{-35}{52} \end{cases}$$

`The intersection is given by` $\overrightarrow{r_1}\left(\frac{19}{26}\right) = \overrightarrow{r_2}\left(\frac{-35}{52}\right)$

$$\boxed{Intersection \left(\frac{33}{26}, \frac{17}{26}\right)}$$

Exercise 14. (36p69) An object is moving in the xy-plane and its position after t seconds is

$$\overrightarrow{r}(t) = \langle t-3, t^2 - 2t \rangle.$$

1. **Answer.**

$$\boxed{\overrightarrow{r}(5) = \langle 2, 15 \rangle, \quad (2, 15)}$$

2. **Answer.**

$$\vec{r}(5+h) = \langle 5+h-2, (5+h)^2 - 2(5+h)\rangle = \langle 3+h, 15+8h+h^2\rangle$$

$$\vec{r}(5+h) - \vec{r}(5) = \langle h, 8h+h^2\rangle$$
$$\frac{1}{h}\left(\vec{r}(5+h) - \vec{r}(5)\right) = \langle 1, 8+h\rangle$$
$$\vec{r'}(5) = \lim_{h\to 0}\frac{1}{h}\left(\vec{r}(5+h) - \vec{r}(5)\right) = \langle 1, 8\rangle$$
$$\boxed{\vec{r}'(5) = \langle 1, 8\rangle}$$

3. **Answer.** It passes through $(1,8)$ if there exists t such that $\vec{r}(t) = \langle 1, 8\rangle$

$$\begin{cases} t-3=1 \\ t^2-2t=8 \end{cases} \qquad \begin{cases} t=4 \\ 4^2-4=8 \end{cases}$$

$$\boxed{\text{It passes through (1,8) at time } t=4}$$

4. **Answer.** It passes through $(3,30)$ if there exists t such that $\vec{r}(t) = \langle 3, 30\rangle$

$$\begin{cases} t-3=3 \\ t^2-2t=30 \end{cases} \qquad \begin{cases} t=6 \\ 6^2-12=24 \neq 30 \end{cases}$$

No t satisfies both equations at the same time.

$$\boxed{\text{It does not pass through (3,30)}}$$

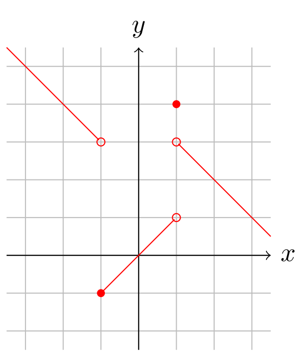
5. **Answer.**

$$\begin{cases} x=t-3 \\ y=t^2-2t \end{cases} \qquad \begin{cases} t=x+3 \\ y=(x+3)^2-2(x+3) \end{cases}$$

$$\boxed{y=(x+3)(x-1)}$$

3 Limits

Exercise 15. (8p 90)

1. **Answer.** 

□

2. **Answer.**

$$\lim_{x\to 1^-} g(x) = 1 \qquad \lim_{x\to 1^+} g(x) = 3 \qquad \lim_{x\to 1} g(x) = DNE$$
$$\lim_{x\to -1^-} g(x) = 3 \qquad \lim_{x\to -1^+} g(x) = -1 \qquad \lim_{x\to -1} g(x) = DNE$$

□

3. **Answer.** The limit does not exist at $x=1$ and $x=-1$ therefore g is not continuous at $x=-1$ and at $x=1$.

□

Exercise 16. Determine the infinite limit

$$\lim_{x\to 5^-} \frac{6}{x-5} = -\infty \qquad \lim_{x\to 5^+} \frac{6}{x-5} = \infty$$
$$\lim_{x\to 5} \frac{6}{x-5} = DNE \qquad \lim_{x\to 2} \frac{3}{(x-2)^2} = \infty$$

Exercise 17. Find the following limits or say why they don't exist.

1. **Answer.** If $x-5>0$, i.e. $x>5$, then $|x-5|=5$.

$$\lim_{x\to 5+} \frac{|x-5|}{x-5} = 1$$

If $x-5<0$, i. e. $x<5$, then $|x-5| = -(x-5)$.

$$\lim_{x\to 5+} \frac{|x-5|}{x-5} = -1$$

The limit from the left and the limit from the right are different, therefore $\lim_{x\to 5} \frac{|x-5|}{x-5}$ does not exist.

□

2. **Answer.** $\lim_{x\to\infty} \frac{x^3+4x^2-6x+7}{x^2-3x+5} = \lim_{x\to\infty} \frac{x^3}{x^2} = \lim_{x\to\infty} x = \infty$.

□

3. **Answer.**

$$\frac{x^3-27}{x^2-9} = \frac{(x-3)(x^2+3x+9)}{(x-3)(x+3)} = \frac{x^2+3x+9}{x+3}$$

$$\frac{\sqrt{4x+4}-\sqrt{5x+1}}{x^2-9} = \frac{(\sqrt{4x+4}-\sqrt{5x+1})(\sqrt{4x+4}+\sqrt{5x+1})}{(x-3)(x+3)} = \frac{-1}{(x+3)(\sqrt{4x+4}+\sqrt{5x+1})}$$

$$\boxed{\lim_{x\to 3} \langle \frac{x^3-27}{x^2-9}, \frac{\sqrt{4x+4}-\sqrt{5x+1}}{x^2-9} \rangle = \langle \frac{9}{2}, \frac{-1}{48} \rangle}$$

4. **Answer.**

$$\begin{aligned}\sqrt{x^2+3x+1}-\sqrt{x^2-4x+3} &= \frac{(\sqrt{x^2+3x+1}-\sqrt{x^2-4x+3})(\sqrt{x^2+3x+1}+\sqrt{x^2-4x+3})}{\sqrt{x^2+3x+1}+\sqrt{x^2-4x+3}} \\ &= \frac{x^2+3x+1-(x^2-4x+3)}{\sqrt{x^2+3x+1}+\sqrt{x^2-4x+3}} \\ &= \frac{7x-2}{\sqrt{x^2+3x+1}+\sqrt{x^2-4x+3}} \\ &= \frac{x\left(7-\frac{2}{x}\right)}{\sqrt{x^2}\left(\sqrt{1+\frac{3}{x}+\frac{1}{x^2}}+\sqrt{1+\frac{4}{x}+\frac{3}{x^2}}\right)}\end{aligned}$$

If $x \to -\infty$, $x < 0$ and $\sqrt{x^2} = -x$

Therefore

$$\boxed{\lim_{x\to-\infty} \sqrt{x^2+3x+1}-\sqrt{x^2-4x+3} = \frac{-7}{2}}$$

Exercise 18. Find the horizontal and vertical asymptotes of

1. $f(x) = \dfrac{\sqrt{x^2-9}}{2x-6}$. **Answer.** domain of f is $(-\infty,-3]\cup(3,\infty)$.

   Possible horizontal asymptotes at $+\infty$ and vertical asymptote at $x = 3$.

   $\lim_{x\to\infty} f(x) = \dfrac{1}{2}$ Therefore $y = \frac{1}{2}$ is a horizontal asymtote at $+\infty$.

   $\lim_{x\to-\infty} f(x) = \dfrac{-1}{2}$ Therefore $y = \frac{-1}{2}$ is a horizontal asymtote at $-\infty$.

   ($\sqrt{x^2} = -x$ when $x < 0$).

   $f(x) = \dfrac{\sqrt{(x-3)(x+3)}}{2(x-3)} = \dfrac{1}{2}\sqrt{\dfrac{x+3}{x-3}}$

   $\lim_{x\to3} f(x) = +\infty$ and $x = 3$ is a vertical asymptote.

□

2. $g(x) = \dfrac{\sqrt{2x+3}-\sqrt{3x}}{x^2-3x}$.

   Answer. Domain= $(0,3)\cup(3,\infty)$

   Possible horizontal asymptote at ∞ and vertical asymptote at $x = 0$ and $x = 3$.

$$\lim_{x\to\infty} g(x) = \lim_{x\to\infty} \frac{\sqrt{x}}{x^2}\frac{\sqrt{2+\frac{3}{x}}-\sqrt{3}}{1-\frac{3}{x}} = 0$$

   $y = 0$ is a horizontal asymptote at ∞.

   If x goes to 0, $x < 1$ and $x^2 < 3x$ therefore $x^2-3x < 0$ and goes to 0.

   The nnumerator goes to $\sqrt{3}$ therefore

$$\lim_{x\to0} g(x) = -\infty$$

   and $x = 0$ is a vertical asymptote.

At 3**:**

$$g(x) = \frac{(\sqrt{2x+3} - \sqrt{3x})(\sqrt{2x+3} + \sqrt{3x})}{x(x-3)(\sqrt{2x+3} + \sqrt{3x})}$$
$$= \frac{2x+3-3x}{x(x-3)(\sqrt{2x+3} + \sqrt{3x})}$$
$$= \frac{-1}{x(\sqrt{2x+3} + \sqrt{3x})}$$
$$\lim_{x\to 3} g(x) = \frac{-1}{3(\sqrt{9} + \sqrt{9})} = \frac{-1}{18}$$

The limit is finite, the graph of g has no vertical asymptote at $x = 3$ but a hole.

□

3. **Answer.** Domain$= (-\infty, -2) \cup (-2, -1) \cup (-1, \infty)$.

   Possible horizontal asymptotes at ∞ and at $-\infty$, possible vertical asymptotes at $x = -2$ and $x = -1$.

   $\lim_{x\to\infty} m(x) = \lim_{x\to\infty} \frac{3x^2}{x^2} = 3$ (same limit at $-\infty$).

   $y = 3$ is a horizontal asymptote at $+\infty$ and at $-\infty$.

   $\lim_{x\to -1} 3x^2 - 3x + 6 = 12 \neq 0$ and $\lim_{x\to -1+} x^2 + 3x + 2 = 0$ and $x^2 + 3x + 2 > 0$ at the right of -1, therefore $x = -1$ is a vertical asymptote and $\lim_{x\to -1^+} = \infty$.

   Again $\lim_{x\to -2} 3x^2 - 3x + 2 = 20 \neq 0$ and $\lim_{x\to -2-} x^2 + 3x + 2 = 0$ and $x^2 + 3x + 2 > 0$ at the left of -2, therefore $x = -2$ is a vertical asymptote and $\lim_{x\to -2^-} = \infty$.

□

4 Continuity

Exercise 19. Let

$$g(x) = \begin{cases} 2x - x^2 & \text{if } 0 \leqslant x \leqslant 2 \\ 2 - x & \text{if } 2 < x \leqslant 3 \\ x - 4 & \text{if } 3 < x < 4 \\ \pi & \text{if } x \geqslant 4 \end{cases}$$

1. For each of the numbers 2, 3, and 4, determine whether g is continuous at that point. **Answer.** Check if the limit from the right, the limit from the left, and the value of the function equal.

   g is continuous at $x = 2$, $x = 3$ and not continuous at $x = 4$

□

2. For each of the numbers 2, 3, and 4, determine whether g is differentiable at that point.

   Answer. Find the one sided limits of $\frac{f(2+h) - f(2)}{h}$ and $\frac{f(3+h) - f(3)}{h}$ to determine whether the limit of $\frac{f(2+h) - f(2)}{h}$ and $\frac{f(3+h) - f(3)}{h}$ exists.

   g is not differentiable at $x = 2$ and at $x-3$. The one sided limits are different at $x = 2$ and $x = 3$. At $x = 4$, g is not continuous therefore g is not differentiable.

□

5 Derivatives

Exercise 20. (54p147) Let $\overrightarrow{r}(t) = \langle \sqrt{x+1}, 2t - 3t^2 \rangle$.

1. Find an tangent vector to the curve given by the graph of $\overrightarrow{r}(t)$ at the point where $t = 3$.
2. Find a vector equation for the tangent line to the curve at the point where $t = 3$.
3. Find a Cartesian equation for the tangent line to the curve at the point where $t = 3$.

Exercise 21. Answer. `To find the derivatives,`

- `Find and simplify` $f(x+h)$
- `Find and simplify` $f(x+h) - f(x)$
- `Find and simplify` $\dfrac{f(x+h) - f(x)}{h}$.
- `The derivtive is the limits when` h `goes to zero.`

`You may see corrected exercises in the week in review 5.`

□

Exercise 22. Answer. `Draw the graph of the function and determine where there are sharp points.`

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