# Review for exam 2

**Exercise 1.** Show that the functions are one-to-one and determine its inverse..

1. See class notes
2. $f(x) = \dfrac{3-x}{5+x}$.

   **Answer.** $f(x_1) = f(x_2)$ `iff`

$$\frac{3-x_1}{5+x_1} = \frac{3-x_2}{5+x_2}$$
$$(5+x_1)(3-x_2) = (5+x_2)(3-x_1)$$
$$-5x_2 + 3x_1 = -5x_1 + 3x_2$$
$$8x_1 = 8x_2$$

$f$ `is one-to-one.`

$f^{-1}(y) = x$ `iff`

$$f(x) = y = \frac{3-x}{5+x}$$
$$y(5+x) = 3-x$$
$$x(1+y) = 3-5y$$
$$\boxed{f^{-1}(y) = x = \frac{3-5y}{1+y}}$$

**Exercise 2.** See class notes.

**Exercise 3.** See your notes.

**Exercise 4.** Find the derivative of

1. See your notes .
2. **Answer.**

$$\boxed{f'(x) = \sec(\mathrm{e}^{\tan x^3})\tan(\mathrm{e}^{\tan x^3})\,\mathrm{e}^{tanx^3}\sec^2(x^3)3x^2}$$

3. **Answer.** $f(x) = \dfrac{x^5 \csc(4x^3)}{e^{x^2-3x}} = x^5 \csc(4x^3)\, e^{-x^2+3x}$

$$\boxed{f'(x) = 5x^4 \csc(4x^3)\, e^{-x^2+3x} + x^5(-\csc(4x^3)\cot(4x^3)(12x^2))\, e^{-x^2+3x} + x^5 \csc(4x^3)\, e^{-x^2+3x}(-2x+3)}$$

**Exercise 5. Answer.**

$$\boxed{f^{(12)}(x) = (-3)^{12}\, e^{-3x}}$$

**Exercise 6.** see class notes

**Exercise 7.** see class notes

**Exercise 8.** Given the curve

$$x(t) = t(t^2-4), \quad y(t) = 3(t^2-4)$$

1. See class notes
2. **Answer.** If $x'(t) = 0$ and $y'(t) \neq 0$, then the curve has a vertical tangent line at $(x(t), y(t))$. $x'(t) = 3t^2 - 4 = 0$ iff $t = \pm\frac{2}{\sqrt{3}}$,

   2 Vertical tangent at the points $(\pm\frac{16}{3\sqrt{3}}, -8)$.

   If $y'(t) = 0$ and $x'(t) \neq 0$ then the curve has a horizontal tangent at $(x(t), y(t))$
   $y'(t) = 6t = 0$ if $t = 0$. 1 horizontal tangent at $(0, -12)$.

□

3. **Answer.** The curve cross itself if there exists $s \neq t$ such that $x(s) = x(t)$ and $y(s) = y(t)$.

$$t(t^2-4) = s(s^2-4), \quad 3(s^2-4) = 3(t^2-4)$$
$$(s^2-4) = (t^2-4) \quad \Rightarrow \quad s = \pm t \quad \Rightarrow \quad s = -t$$
$$-t(t^2-4) = t(t^2-4) \quad \Rightarrow \quad t^2 - 4 = 0 \quad \Rightarrow \quad t = \pm 2 = -s$$

1 point $(x(2), y(2)) = (0, 0)$

□

**Exercise 9.** Find the second derivative $y''$ if

$$e^{xy} = x - y$$

**Answer.**

$$e^{xy}(xy' + y) = 1 - y'$$

$e^{xy} = x - y$ therefore

$$(x-y)(xy'+y) = 1 - y'$$
$$y'(x(x-y)+1) = 1 - y(x-y)$$
$$y'(x^2 - xy + 1) = 1 - xy + y^2$$
$$y''(x^2 - xy + 1) + y'(2x - xy' - y) = -y'x - y + 2yy'$$
$$y''(x^2 - xy + 1) = -3xy' + 3yy' - y + xy'^2$$
$$y''(x^2 - xy + 1) = \left(\frac{1 - xy + y^2}{1 - xy + x^2}\right)(-3x + 3y) - y + x\left(\frac{1 - xy + y^2}{1 - xy + x^2}\right)^2$$
$$y'' = \frac{1}{1 - xy + x^2}\left(\left(\frac{1 - xy + y^2}{1 - xy + x^2}\right)(-3x + 3y) - y + x\left(\frac{1 - xy + y^2}{1 - xy + x^2}\right)^2\right)$$

□

**Exercise 10. Answer.**

$$\boxed{f^{(23)}(x) = -2^{23}\cos(2x)}$$

**Exercise 11. Answer.** Intersection when $x = 2y$ and $x^2 + y^2 = 5$.

$x = 2y$ and $5y^2 = 5$.

$y = \pm 1$, 2 intersection points $(2, 1)$ and $(-2, -1)$.

Slope of the circle at $(2, 1)$ and at $(-2, -1)$ is given by $y'$

$$2x + 2y'y = 0$$

At $(2, 1)$, slope is $y' = -\frac{x}{y} = -2$.

At $(-2, -1)$, slopw is $y' = -\frac{x}{y} = -2$

The slope of the line is $1/2$. Therefore the curves are orthogonales.

□

**Exercise 12. Answer.**

$$-2\sin(x - y)(1 - y') = e^{x+y}(1 + y')$$

At $(\pi/6, -\pi/6)$, $-2\sin(\pi/3)(1 - y') = (1 + y')$

$$-\sqrt{3}(1 - y') = 1 + y'$$

`Slope:` $y' = \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$.

$$\boxed{y+\pi/6 = \frac{\sqrt{3}+1}{\sqrt{3}-1}(x-\pi/6)}$$