
Review for exam 3

Exercise 1. Find $f(x)$ for

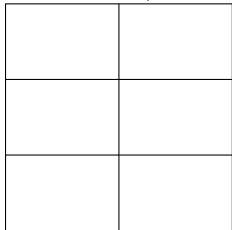
1. $f'(x) + x - \sqrt[4]{x}, f(1) = 1.$
2. $f'(x) = \frac{3 + 2x}{\sqrt{x}}, f'(1) = 0.$
3. $f''(x) = 5 \sin x - 3 \cos x, f'(x) = 1, f(x) = -3.$

Exercise 2. Find the vector $\vec{r}(t)$ that gives the position of a particle at time t having the acceleration $\vec{a}(t) = \langle 2t, 3 \rangle, \vec{v}(0) = \langle 1, -1 \rangle$ and initial position $(1, 2).$

Exercise 3. Find the point on the hyperbola $xy = 8$ that is the closest to the point $(3, 0).$

Exercise 4. If 1200cm^2 of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

A farmer with 600ft of fencing wants to enclose a rectangular area and then divide it into 6 equal pens (see figure below). What is the largest possible area of the 6 pens.



Exercise 5. A piece of wire of 10 inches long is cut into two pieces. One piece is bent into a square and the other is bent into an equilateral triangle. How should the wire be cut so that the total area enclosed is maximum? minimum?

Exercise 6. Let $f(x) = \frac{\ln x}{\sqrt{x}}$

1. Find the intervals on which f is increasing or decreasing?
2. Find the local minimum, local maximum values of $f.$
3. Find the intervals of concavity and the inflection points.

Exercise 7. Let $f(x) = x\sqrt{x+1}.$

1. Find the intervals on which f is increasing or decreasing?

2. Find the local minimum, local maximum values of f .
3. Find the intervals of concavity and the inflection points.

Exercise 8. Let $f(x) = 2x + \cot x$, $0 < x < \pi$.

1. Find the intervals on which f is increasing or decreasing?
2. Find the local minimum, local maximum values of f .
3. Find the intervals of concavity and the inflection points.

Exercise 9. $f(x) = \sqrt{x^2 + 1} - x$.

1. Find the asymptotes.
2. Find the intervals on which f is increasing or decreasing?
3. Find the local minimum, local maximum values of f .
4. Find the intervals of concavity and the inflection points.

Exercise 10. $f(x) = x^5(x - 1)^6$

1. Find the intervals on which f is increasing or decreasing?
2. Find the local minimum, local maximum values of f .
3. Find the intervals of concavity and the inflection points.

Exercise 11. Find the absolute maximum and absolute minimum values of f on the given interval.

1. $f(x) = \frac{x}{x+1}$, $[1, 2]$
2. $f(x) = \sqrt{9 - x^2}$, $[-1, 2]$.
3. $f(x) = x^2 - 2x + 2$, $[0, 3]$. item $f(x) = \frac{\cos x}{2 + \sin x}$, $[0, 2\pi]$.

Exercise 12. Find the critical numbers of $f(x) = |x^2 - 1|$.

Exercise 13. Find the critical numbers of $f(x) = \sqrt[3]{x^2 - x}$.

Exercise 14. Find the following limits

1. $\lim_{x \rightarrow 0} \frac{6^x - 5^x}{x}$.

2. $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3}$.

3. $\lim_{x \rightarrow 0} \frac{\sin x}{e^x}$.

4. $\lim_{x \rightarrow 0} \left(\frac{2x + \text{Arcsin } x}{3x - \text{Arctan } x} \right)$.

5. $\lim_{x \rightarrow \infty} e^{-x} \ln x$.

6. $\lim_{x \rightarrow 0} \sqrt{x} \ln(x)$.

7. $\lim_{x \rightarrow 1} \left(\frac{1}{\ln x} - \frac{1}{x-1} \right)$.

8. $\lim_{x \rightarrow 0} (\sin x)^{\tan x}$.

9. $\lim_{x \rightarrow \infty} \left(1 + \frac{3}{x} \right)^{5x}$.

10. $\lim_{x \rightarrow \infty} (e^x + x)^{1/x}$.

Exercise 15. Simplify the expressions

1. $\text{Arctan} \left(\tan \left(\frac{6\pi}{5} \right) \right)$

2. $\cos(\text{Arcsin } 0.1)$.

3. $\sec(\text{Arctan } 2)$.

4. $\tan(\text{Arccos } 0.5)$

Exercise 16. Find the derivative of $f(x) = \text{Arctan}(\sin x)$ and simplify.

Exercise 17. Find the domain and the derivative of $f(x) = \text{Arccos}(\text{Arcsin } x)$.

Exercise 18. Find the domain and the derivative of $\text{Arccos}(e^x)$,

Exercise 19. Find the domain and the derivative of $\text{Arcsin}((1 - x^2))$

Exercise 20. Find $\lim_{x \rightarrow 0^+} \text{Arctan}(\ln 3x)$.

Exercise 21. Find $\lim_{x \rightarrow \infty} \text{Arccos} \left(\frac{2x-1}{2x+1} \right)$.

Exercise 22. Find $\lim_{x \rightarrow \infty} \text{Arcsin} \left(\frac{x-1}{x+1} \right)$.

Exercise 23. Polonium-210 has half life of 140 days.

1. If a sample has a mass of 200mg, find a formula for the mass that remains after t days.
2. Find the mass after 100 days.
3. When will the mass be reduced to 10mg?

Exercise 24. A bacteria culture starts with 400 bacteria and after 3 hours there are 3200 bacteria.

1. Find an expression for the number of bacteria after t hours.
2. Find the number of bacteria after 4 hours.
3. When will the number of bacteria reach 10,000?

Exercise 25. A roast turkey is taken from an oven when its temperature has reached 185°F and is placed on a table in a room where the temperature is 75°F.

1. If the temperature of the turkey is 150°F after half an hour, what is the temperature after 45 min?
2. When will the turkey have cooled to 100°?

Exercise 26. Evaluate

1. $\log_2(64), \log_8(32)$.
2. $\log_3(9^{\sqrt{3}})$
3. $e^{\ln(2)} + \ln(e^{\sqrt{2}})$

Exercise 27. Find the domain and solve the equations

1. $\ln(2e^x - 1) = 3$
2. $\ln(x) + \ln(x - 1) = \ln(.5)$
3. $\log_2(\log_3(\log_4(x))) = 0$
4. $\ln\left(\frac{x-2}{x-1}\right) = 1 + \ln\left(\frac{x-3}{x-1}\right)$

Exercise 28. Find the derivative of

1. $f(x) = \sqrt{x} \ln(x)$ and state the domain.
2. $g(x) = \ln\left(\frac{x^2 \sin x}{(\text{Arctan } x)\sqrt{x^2 + 3}}\right)$

3. $f(x) = \ln(\sec x + \tan x)$ and simplify.

4. $f(x) = x^{\ln(x)} - (\ln x)^x$.

5. $h(x) = (\sin x)^{\cos x}$

Exercise 29. Using the logarithmic differentiation, find the derivative of

$$f(x) = \sqrt{\frac{(\cos^6 x)(e^{3x^2})\sqrt{x^5 - 1}}{(2x + 1)x^{2/5}}}$$

Exercise 30. Find y' if $y = \ln(x^2 + y^2)$.