Section 3.11, Linear and Quadratic approximation

Definition: Let f be a differentiable function at $x = a$, the function

$$L(x) = f(a) + f'(a)(x - a)$$

is the linearization of f at a. When x is near a, the value $L(x)$ is called a linear approximation, or tangent line approximation of f at a.

Exercise 1. Find the linearization of the function $f(x) = \frac{1}{x}$ at $x = 10$.
Find an approximation of $\frac{1}{10.1}$.

Exercise 2. Find a linear approximation of the function $f(x) = \sqrt[3]{1+x}$ at $x = 0$ and use it to approximate the number $\sqrt[3]{0.95}$.

Exercise 3. Use a linearization to find an approximation for the numbers

$$\sqrt{36.1}, \quad \sin(59^\circ), \quad (1.97)^3$$

Definition: Let f be a two times differentiable function at $x = a$, the function

$$Q(x) = f(a) + f'(a)(x - a) + \frac{f''(a)}{2}(x - a)^2$$

is called a quadratic approximation of f at a.

Exercise 4. Find a quadratic approximation of $f(x) = \cos x$ at $x = 0$

Exercise 5. Find a quadratic approximation of $f(x) = 3x^2 - 2x + 1$ at $x = 0$.

Exercise 6. Let f be a function such that $f(1) = 2$ and $f'(x) = \sqrt{x^3 + 1}$.

1. Use a linear approximation to estimate the value of $f(1.1)$.

2. Do you think the real value of $f(1.1)$ is less or greater than your estimate?

3. Use a quadratic approximation to estimate the value of $f(1.1)$.