Texas A&M University

Chapter 4, section 4, Chain rule

1 General Power rule

Theorem: Let $u(x)$ be a differentiable function and n be a real number ($n \neq 0$).

$$(u^n(x))' = nu(x)^{n-1}u'(x).$$

Exercise 1. Using the general power rule, find the derivative of

1. $d(x) = (3x + 5)^2$.

2. $h(t) = 2\sqrt{3t - 5}$.

3. $i(z) = \dfrac{1}{z^2 + 1}$.

2 Chain Rule

Theorem: If $y = f(u)$ and $u = g(x)$ define the composite function $y = m(x) = f(g(x))$, then

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u}\frac{\mathrm{d}u}{\mathrm{d}x} \qquad m'(x) = f'(g(x))g'(x).$$

Exercise 2. Use the chain rule to find the derivatives of

1. $k(x) = \cos(3x + 1)$.

2. $v(x) = \big(\sin(x^2)\big)^6$.

3. $w(x) = \cot(\sqrt[3]{1 + x^3})$.

4. (44p193) $y(x) = \sqrt{x + \sqrt{x + \sqrt{x}}}$

5. $l(x) = ((x^3 + x + 1)^5)\sqrt{x^2 - 3x + 10}$.

6. $m(x) = \sqrt{\sin(\cos^3 x)}$.

Exercise 3. (58p193) Find the x-coordinates of all the points on the curve

$$y = \sin 2x - 2\sin x$$

at which the tangent line is horizontal.

Exercise 4. Suppose that $u(x) = f(g(x))$ and

$$f(0) = 3, \quad f'(0) = 2, \quad f(1) = 5, \quad f'(1) = -4, \quad g(0) = 1, \quad g'(0) = 7.$$

Exercise 5. Find an equation to the tangent line to the curve $y = \tan(\pi x^3/4)$ at the point $(1, 1)$. Find $u'(0)$.