Section 3.9, Slopes and tangents of parametric curves

Theorem: Suppose a curve is given by the parametric equations

$$x = x(t), \quad y = y(t)$$

The slope of the tangent line is given by

$$slope = \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{rise}{run} = \frac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}}.$$

Or,

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}x}{\mathrm{d}t}$$

.

Exercise 1. Find an equation of the tangent to the curve

$$x(t) = t^2 + t, \quad y(t) = \sqrt{t}$$

at the point corresponding to the given value of the parameter $t = 4$.

Exercise 2. Find an equation of the tangent line to the curve

$$\overrightarrow{r}(t) = \langle 5\cos t, 5\sin t \rangle$$

at the point $(3, 4)$.

Exercise 3. At what point does the curve

$$x(t) = 1 - 2\cos^2 t, \quad y(t) = (\tan t)(1 - 2\cos^2 t)$$

cross itself?

Find equations of both tangents at that point.

Exercise 4. Find equations of the tangents to the curve

$$x(t) = 3t^2 - 1, \quad y(t) = 2t^3 + 3$$

that pass through $(2, 5)$.