Texas A&M University

Section 4.1 Exponential function and their derivatives

Theorem: If $a > 0$, and $a \neq 1$ the function f(x)=a^x is continuous on $\mathbb{R}$.
If $0 < a < 1$, a^x is decreasing,

$$\lim_{x\to\infty} a^x = 0 \qquad \lim_{x\to-\infty} a^x = \infty$$

If $a > 1$, a^x is increasing,

$$\lim_{x\to\infty} a^x = \infty \qquad \lim_{x\to-\infty} a^x = 0$$

f is differentiable and $f'(x) = a^x f'(0)$.
e is the number such that $\frac{\mathrm{d}\,\mathrm{e}^x}{\mathrm{d}x} = \mathrm{e}^x$ i.e. $f'(0) = 1$.

Exercise 1. Find the limits

$$\lim_{x\to\infty} (1.1)^{2x-1}, \qquad \lim_{x\to 0^+} -3^{1/x}$$

$$\lim_{x\to\infty} \frac{2\,\mathrm{e}^{3x} - 5\,\mathrm{e}^{-3x}}{\mathrm{e}^{3x} - 4\,\mathrm{e}^{-3x}}, \qquad \lim_{x\to\pi/2^-} \frac{2}{1+\mathrm{e}^{\tan x}}.$$

Exercise 2. Differentiate the functions

$$f(x) = \mathrm{e}^{\tan x}, \qquad g(x) = \tan\left(\mathrm{e}^{3x^2-1}\right), \qquad h(x) = x^{\mathrm{e}} - e^x$$

Exercise 3. Find an equation to the tangent line to the curve

$$y = x^2\,\mathrm{e}^{-x}$$

at the point $(1, 1/\mathrm{e})$.

Exercise 4. Find an equation of the tangent line to the curve

$$2\,\mathrm{e}^{xy} = x + y$$

at the point $(0, 2)$.