Texas A&M University

Section 4.2 Inverse functions

Definition: A function f is one-to-one if $f(x_1) = f(x_2)$ implies $x_1 = x_2$.

Exercise 1. Determine whether the functions $f(x) = \dfrac{1}{x-1}$ and $g(x) = x^2 - 2x + 5$ are one-to-one.

Definition: Let f be a one-to-one function with domain A and range B. Then its inverse function f^{-1} has domain B and range A and is defined by

$$f^{-1}(y) = x \Longleftrightarrow f(x) = y$$

for any y in B.

Exercise 2. Find the inverse function of $f(x) = \dfrac{3x-2}{x-1}$ and $g(x) = 5 - 4x^3$.

Exercise 3. Find the inverse function of $f(x) = x^2 + x, \quad x \geqslant \frac{-1}{2}$.

> **Theorem:** Let f be a differentiable function and g be the inverse function of f, g is differentiable at y if $f'(x) \neq 0, quady = f(x)$ and $g'(y) = \frac{1}{f'(x)}$.

Exercise 4. Let $f(x) = 1 + x + x^3$. Find $g'(1)$ and $g'(3)$ with $g = f^{-1}$.

Exercise 5. Let $f(x) = 3 + x + \mathrm{e}^x$. Find $g'(4)$ with $g = f^{-1}$.

Exercise 6. Suppose g is the inverse function of a differentiable function f and $G(x) = 1/g(x)$. If $f(3) = 2$ and $f'(3) = \frac{1}{9}$, find $G'(2)$.