Section 4-6, Inverse trigonometric functions

Exercise 1. Which of the following functions is one-to-one?

$$f_1 : \mathbb{R} \longrightarrow [-1,1] \qquad f_2 : [0,\pi] \longrightarrow [-1,1] \qquad f_3 : [-\frac{\pi}{2},\frac{\pi}{2}] \longrightarrow [-1,1]$$
$$x \longmapsto \sin x \qquad x \longmapsto \sin x \qquad x \longmapsto \sin x$$

Definition: $\sin^{-1}$ or Arcsin is the inverse function of

$$\sin : [-\pi/2, \pi/2] \longrightarrow [-1,1].$$

Domain of Arcsin:

Range of Arcsin:

For $x \in [-\pi/2, \pi/2]$, $\sin^{-1}(\sin x) =$

For $y \in [-1,1]$, $\sin\left(\sin^{-1} y\right) =$

$\text{Arcsin}'(x) = \frac{\mathrm{d}}{\mathrm{d}x}\left(\sin^{-1}\right)(x) =$

Definition: $\cos^{-1}$ or Arccos is the inverse function of

$$\cos : [0,\pi] \longrightarrow [-1,1].$$

Domain of Arccos: $[-1,1]$.

Range: $[0,\pi]$

For $x \in [0,\pi]$, $\cos^{-1}(\cos x) = x$

For $y \in [-1,1]$, $\cos(\cos^{-1} y) = y$

$\text{Arccos}'(x) = \frac{\mathrm{d}}{\mathrm{d}x}\left(\cos^{-1}\right)(x) = \frac{-1}{\sqrt{1-x^2}}$

Definition: $\tan^{-1}$ or Arctan is the inverse function of

$$\tan :]-\pi/2, \pi/2[\longrightarrow \mathbb{R}.$$

Its domain is $\mathbb{R}$.

Its range is $]-\pi/2, \pi/2[$.

For $x \in]\pi/2, \pi/2[$, $\tan^{-1}(\tan x) = x$

For $y \in \mathbb{R}$, $\tan(\tan^{-1} y) = y$

$\text{Arctan}'(x) = \frac{\mathrm{d}}{\mathrm{d}x}\left(\tan^{-1}\right)(x) = \frac{1}{1+x^2}$.

Exercise 2. Find the values of

1. $\tan(\cos^{-1}(0.5))$

2. $\text{Arctan}\left(\tan \frac{4\pi}{3}\right)$.

3. $\cos^{-1}(\cos 3\pi)$

4. $\cos(\tan^{-1}(2))$

5. $\tan\left(\sin\left(\frac{3}{4}\right)\right)$

Exercise 3. Simplify $\sin(2\cos^{-1} x)$.

Exercise 4. Find the derivative of $\cos^{-1}(\sin x)$ for $x \in [0, \pi/2]$.

Exercise 5. Find the derivative of $f(x) = \sin^{-1}(x^2)$.

Exercise 6. Show that for $x \in [0, \pi/2], \text{Arcsin}\, x + \text{Arccos}\, x = \pi/2$.

Exercise 7. Find the derivative of $\tan^{-1}(\sqrt{x^5 + 4})$.