Section 4.8, L'hospital's rule

Theorem L'Hospital's rule: Suppose that f and g are differentiable and $g'(x) \neq 0$ on an open interval I that contains a (except possibly at a). Suppose that

$$\lim_{x\to a} f(x) = 0 \quad \text{and} \quad \lim_{x\to a} g(x) = 0$$

or that

$$\lim_{x\to a} f(x) = \pm\infty \quad \text{and} \quad \lim_{x\to a} g(x) = \pm\infty$$

Then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

Remark:

Exercise 1. Calculate the limit

$$\lim_{x\to 2} \frac{x^3 - 12x + 16}{x^3 - 3x^2 + 4}.$$

Exercise 2. Find $\lim_{x\to 0^+} \frac{\ln x}{\sqrt{x}}$.

Exercise 3. Find the limit of $\frac{\cos x - 1}{x^2}$ when x goes to 0.

Exercise 4. $\lim_{\theta\to 0}\dfrac{\tan^{-1}\theta-\theta}{\theta^3}=$

Exercise 5. $\lim_{x\to 0}\dfrac{\ln(1-x)+x+\frac{x^2}{2}}{x^3}=$

Exercise 6. Calculate the limit

$$\lim_{x\to 0}\left(\frac{1}{\ln(1+x)}-\frac{1}{x}\right).$$

Exercise 7. Evaluate $\lim_{x\to 0}\dfrac{\mathrm{e}^{x^2}-1-x^2}{x^3}$.

Exercise 8. Find $\lim_{x\to+\infty}(1+2x)^{1/x}$.

Exercise 9. Evaluate $\lim_{x\to+\infty}\left(1-\dfrac{1}{3x}\right)^{2x}$.

Exercise 10. Sometimes, this rule does not help : Use L'Hospital's rule to find the limit

$$\lim_{x\to+\infty} \frac{x}{\sqrt{x^2+1}}$$

Exercise 11. $\lim_{x\to 0^+} \frac{(\sin(3x))^6}{(\tan 4x^2)^3}$.