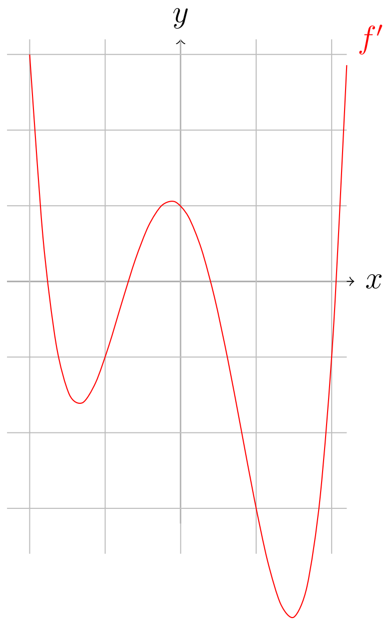
Texas A&M University

Section 5.1

Theorem: Let f bea 2 times differentiable function on I
If $f'(x) > 0$ on an interval $J \subset I$, then f is increasing on J.
If $f'(x) < 0$ on an interval $J \subset I$, then f is decreasing on J
If $f''(x) > 0$ on an interval $J \subset I$, the f is concave upward on J.
If $f''(x) < 0$ on an interval $J \subset I$, then f is concave downward on J.

Exercise 1. The graph of f' is shown below:



1. On what intervals is f increasing? decreasing?

2. At what values of x does f has a local maximum? a local minimum ?

3. On what interval is f concave upward?

4. On what insterval is f convave downward?

Exercise 2. Sketch the graph of a function that satisfies the conditions

$$f'(-1) = 0, \qquad f'(1) \text{ does not exist}$$

$$f'(x) < 0 \text{ if } |x| < 1, \qquad f'(x) > 0 \text{ if } |x| > 1$$

$$f(-1) = 4, \qquad f(1) = 0$$

$$f''(x) < 0 \text{ if } x \neq 1$$

Exercise 3. If $f'(x) = x(1 - x^2)$ for $-3 \leqslant x \leqslant 2$, in which interval, the function is increasing?

Exercise 4. Determine the intervals where f is increasing, the intervals where f is decreasing, the critical points. Say which critical point is a local minimum, a local maximum, or none of these.

1. $f(x) = (x - 1)^3$.

2. $f(x) = 2x^3 - 3x^2 - 6x + 7$.

3. $f(x) = \dfrac{4}{x} + x$.

4. $f(x) = \dfrac{x^2}{x - 1}$.

> **Definition:** The critical values x of a function f are points of the domain where $f'(x) = 0$ or, where $f'(x)$ does not exist.

Exercise 5. Find all the critical values of $f(x) - 3x^4 - 8x^3 + 6x^2$.

Exercise 6. Suppose the only information we have about the function f is that $f'(x) = x^4 - 2x^2 + 1$.

Over which intervals is the curve $y = f(x)$ concave up?

Exercise 7. Find the inflection points of $f(x) = x^5 - 5x^4$.

Exercise 8. Find the intervals where the graph is concave upward, the intervals where the graph is concave downward, and the inflexion points of the following functions.

1. $f(x) = 2x^6 - 5x^4 + 13x - 9$

2. $h(x) = 2x^3 - 3x^2 - 6x + 7$

3. g is a function such that $g''(x) = x^2 - 25$.