Section 5.2

Definition: A function has an absolute maximum at c if

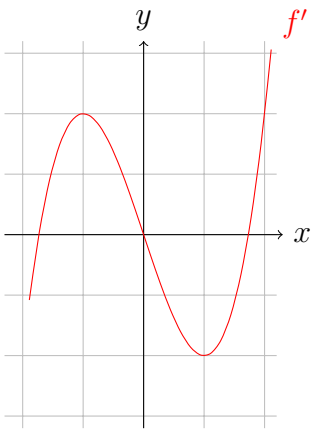
$$f(c) \geqslant f(x)$$

for all x in the domain of f.
$f(c)$ is called the maximum value.
A function f has an absolute minimum at c if

$$f(c) \leqslant f(x)$$

for all x in the domain of f.
$f(c)$is the minimum value.
A function f has a local maximum at c if $f(c) \geqslant f(x)$ for all x in an open interval containing c.
A function f has a local minimum at c if $f(c) \leqslant f(x)$ for all x in an open interval containing c.

Exercise 1. Find the local maximum, local minimum, absolute maximum, absolute minimum of the function f is defined by its graph below:



Exercise 2. Find the absolute and local maximum, and minimum values of f.Begin by sketching the graph.

$$f(x) = 1 - x^2, \qquad -2 \leqslant x \leqslant 1$$

$$f(x) = \begin{cases} x^2 & \text{if } -1 \leqslant x < 0 \\ 2 - x^2 & \text{if } 0 \leqslant x \leqslant 1 \end{cases}$$

The extreme value Theorem: If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum $f(c)$ and an absolute minimum $f(d)$ at some c and d in $[a, b]$.

Fermat's Theorem: If f has a local maximum or a local minimum at c and if $f'(c)$ exists, then $f'(c) = 0$

Remark:

Definition: A critical number of a function f is a number c in the domain of f such that $f'(c) = 0$ or $f'(c)$ does not exist. A critical value of f is $f(c)$ if c is a critical number.

Exercise 3. Find the critical number of

1. $f(x) = t^4 + 4t^3 + 2t^2$

2. $g(x) = |3x - 1|$

3. $h(x) = x\,\mathrm{e}^{3x}$

4. $j(x) = x^{4/5}(x-4)^2$

5. $k(x) = x \ln x$

Theorem: If f has a local extrema at c, then c is a critical number.

Exercise 4. Find the absolute maximum, absolute minimum values of the functions

1. $f(x) = 3x^5 - 5x^3 - 1$ on the interval $[-2, 2]$

2. $f(x) = \dfrac{x}{x+1}$ on the interval $[1, 2]$.

3. $f(x) = \sin x + \cos x$ on the interval $[0, \pi/3]$.