Section 5.3

Exercise 1. Given the function $f(x) = x^{(1/3)}(x+3)^{(2/3)}$

Domain of $f = \mathbb{R}$.

1. Find the intervals on which f is increasing or decreasing.

$$f'(x) = \frac{1}{3}x^{(-2/3)}(x+3)^{(2/3)} + \frac{2}{3}(x^{(1/3)}(x+3)^{(-1/3)} = \frac{1}{3}x^{(-2/3)}(x+3)^{(-1/3)}(x+3+2x)$$

$$= \frac{1}{3}x^{(-2/3)}(x+3)^{(-1/3)}(3x+3) = x^{(-2/3)}(x+3)^{(-1/3)}(x+1)$$

critical numbers : $x=-3$, $x=-1$, $x=0$

$$x^{-2/3} = \frac{1}{\sqrt[3]{x^2}} > 0$$

	$(-\infty,-3)$	$(-3,-1)$	$(-1,0)$	$(0,\infty)$
$x^{-2/3}$	+	+	+	+
$(x+3)^{-1/3}$	−	+	+	+
$x+1$	−	−	+	+
$f'(x)$	+	−	+	+
f	$\nearrow$	$\searrow$	$\nearrow$	$\nearrow$

f is increasing on $(-\infty,-3)$ and on $(-1,\infty)$

f is decreasing on $(-3,-1)$

2. Find the local maximum and minimum values of f.

local minimum at $x=-1$

local minimum value $f(-1) = -\sqrt[3]{4}$

local maximum at $x=-3$

local maximum value $f(-3) = 0$

3. Find the intervals of concavity and the inflection points.

$$f''(x) = x^{(-2/3)}(x+3)^{(-1/3)} + (x+1)\left(\frac{-2}{3}\right)x^{(-5/3)}(x+3)^{(-1/3)} + (x+1)\,x^{(-2/3)}\left(\frac{-1}{3}\right)(x+3)^{(-4/3)}$$

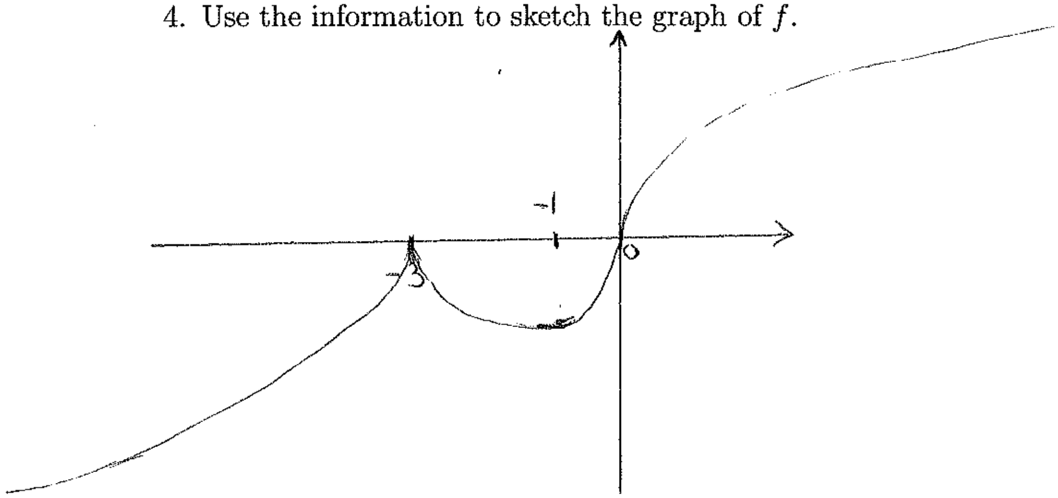
$$= \frac{x^{(-5/3)}(x+3)^{(-4/3)}}{3}\Big(3x(x+3) - 2(x+1)(x+3) - (x+1)(x)\Big)$$

$$= \frac{x^{(-5/3)}(x+3)^{(-4/3)}}{3}\left(3x^2 - 2x^2 - x^2 + 9x - 8x - x - 6\right)$$

$$= \frac{-2}{x^{5/3}(x+3)^{4/3}}$$

	$(-\infty,-3)$	$(-3,0)$	$(0,\infty)$
$x^{5/3}$	$-$	$-$	$+$
$(x+3)^{4/3}$	$+$	$+$	$+$
-2	$-$	$-$	$-$
$f''(x)$	$+$	$+$	$-$
	concave up	concave up	concave down.

4. Use the information to sketch the graph of f.



Exercise 2. Given $f(x) = 2\sin x + sin^2 x$ for x in $[0, 2\pi]$.

1. Find the intervals on which f is increasing or decreasing.

$f'(x) = 2\cos x + 2\sin x\cos x = 2\cos x(1+\sin x)$

critical numbers $\frac{\pi}{2}, \frac{3\pi}{2}$

	$[0,\frac{\pi}{2})$	$(\frac{\pi}{2},\frac{3\pi}{2})$	$(\frac{3\pi}{2},2\pi)$
$\cos x$	$+$	$-$	$+$
$1+\sin x$	$+$	$+$	$+$
$f'(x)$	$+$	$-$	$+$
f	$\nearrow$	$\searrow$	$\nearrow$

f is increasing on $[0,\frac{\pi}{2})$ and on $(\frac{3\pi}{2}, 2\pi]$.

f is decreasing on $(\frac{\pi}{2},\frac{3\pi}{2})$

2. Find the local maximum and minimum values of f.

local maximum at $x = \frac{\pi}{2}$.

local maximum value : $F(\frac{\pi}{2}) = 2 + 1^2 = 3$

local minimum at $x = \frac{3\pi}{2}$

local minimum value = $F(\frac{3\pi}{2}) = -2 + 1 = -1$

3. Find the intervals of concavity and the inflection points.

$F''(x) = -2\sin x + 2\cos^2 x - 2\sin^2 x = -2\sin x + 2(1-\sin^2 x) - 2\sin^2 x$
$= -2(\sin x + 2\sin^2 x - 1)$

IF $X = \sin x$ $\quad F''(x) = -2(X + 2X - 1) = -2(X+1)(2X-1) = -2(\sin x + 1)(2\sin x - 1)$

$F''(x) = 0 \quad x = \frac{3\pi}{2}, \quad x = \frac{\pi}{6}, \quad x = \frac{5\pi}{6}$

	$[0, \frac{\pi}{6})$	$(\frac{\pi}{6}, \frac{5\pi}{6})$	$(\frac{5\pi}{6}, \frac{3\pi}{2})$	$(\frac{3\pi}{2}, 2\pi]$
-2	−	−	−	−
$\sin x + 1$	+	+	+	+
$2\sin x - 1$	−	+	−	−
$F''(x)$	+	−	+	+
F	concave up	concave down	concave up	

F concave upward on $[0, \frac{\pi}{6}]$, on $(\frac{5\pi}{6}, 2\pi]$

F is concave downward on $(\frac{\pi}{6}, \frac{5\pi}{6})$

inflection point at $x = \frac{\pi}{6}$, $x = \frac{5\pi}{6}$

Exercise 3. Given $f(x) = (x^2 - 1)^4$.

1. Find the intervals on which f is increasing or decreasing.

$f'(x) = 4(2x)(x^2-1)^3 = 8x(x-1)^3(x+1)^3$

critical numbers $x = 0$, $x = 1$, $x = -1$

	$(-\infty, -1)$	$(-1, 0)$	$(0, 1)$	$(1, \infty)$
$8x$	−	−	+	+
$(x-1)^3$	−	−	−	+
$(x+1)^3$	−	+	+	+
F'	−	+	−	+
F	↘	↗	↘	↗

F increasing on $(-1, 0)$ and on $(1, \infty)$ $\qquad$ F is decreasing on $(-\infty, -1)$, and on $(0, 1)$

2. Find the local maximum and minimum values of f.

local minimum at $x = -1$; $f(-1) = 0$

local minimum value : $f(1) = 0$ at $x = 1$

local maximum value $f(0) = 1$ at $x = 0$

3. Find the intervals of concavity and the inflection points.

$$f''(x) = 8(x^2-1)^3 + 24x(2x)(x^2-1)^2$$
$$= 8(x^2-1)^2(x^2-1+6x^2)$$
$$= 8(x^2-1)^2(7x^2-1)$$

$f''(x) = 0$ if $x = 1$, $x = -1$ $x = \frac{-1}{\sqrt{7}}$, $x = \frac{1}{\sqrt{7}}$

	$(-\infty, -1)$	$\left(-1, -\frac{1}{\sqrt{7}}\right)$	$\left(\frac{-1}{\sqrt{7}}, \frac{1}{\sqrt{7}}\right)$	$\left(\frac{1}{\sqrt{7}}, 1\right)$	$(1, \infty)$
$8(x^2-1)^2$	+	+	+	+	+
$7x^2-1$	+	+	−	+	+
$f''(x)$	+	+	−	+	+

f

concave upward | concave downward | concave upward.

inflection point at $x = +\frac{1}{\sqrt{7}}$, and at $x = \frac{-1}{\sqrt{7}}$