Section 5.7

Definition: A function F is called an antiderivative of f on an interval I if $F'(x) = f(x)$ for all x in I.

Exercise 1. Find an antiderivative of $f(x) = 2x + 3$

Find a second antiderivative.

Theorem: If F is an antiderivative of f on an interval I, then the most general antiderivative of f on I is $F(x) + C$ where C is an arbitrary constant.

Exercise 2. Find the most general antiderivative of the functions

1. $f(x) = 3x^2 + 4x - 1$

2. $g(x) = \sqrt{x} + \sqrt[3]{x} + \dfrac{1}{x^2} + \dfrac{1}{x^3} + \dfrac{1}{x}$.

3. $h(x) = \dfrac{x^2 - 2x}{\sqrt{x}}$

4. $j(x) = \sin t + \sec^2 t - 3\cos t + 2\sec x \tan x + e^x$.

Theorem:

Function	*Antiderivative*
$x^n (n \neq -1)$	$\dfrac{x^{n+1}}{n+1} + C$
$\dfrac{1}{x}$	$\ln \lvert x \rvert + C$
e^x	$e^x + C$
a^x	$\dfrac{a^x}{\ln a} + C$
$\sin x$	$-\cos x + C$
$\cos x$	$\sin x + C$
$\sec^2 x = 1 + \tan^2 x$	$\tan x + C$
$\csc^2$	$-\cot x + C$
$\sec x \tan x$	$\sec x + C$
$\dfrac{1}{1+x^2}$	$\text{Arctan}\, x + C$
$\dfrac{1}{\sqrt{1-x^2}}$	$\text{Arcsin}\, x + C$

Exercise 3. Find $f(x)$ such that $f'(x) = x^{-2} x > 0, f(1) = 0$

Exercise 4. Find $f(x)$ such that $f''(x) = x^2 + 3\cos x, \quad f(0) = 2, \quad f'(0) = 3.$

Exercise 5. A particle is moving with

$$a(t) = t^2 - t, \quad s(0) = 0, \quad v(0) = 5$$

Find the position of the particle $s(t)$.

Exercise 6. A car is traveling at 50mi/h when the brakes are fully applied, producing a constant deceleration of 40ft/s^2. What is the distance covered before the car comes to a stop.

Exercise 7. Find the vector function that describe the velocity and the position of the particle that has an acceleration

$$a(t) = 2\mathbf{i} + 2t\mathbf{j}, \quad v(0) = \mathbf{i} - \mathbf{j}, \quad s(0) = \mathbf{i} + \mathbf{j}$$