Section 6.1

Definition: Let a_1, a_2, $\cdots$, a_n n real number. If $m \leqslant n$,

$$\sum_{i=m}^{n} a_i = a_m + a_{m+1} + \cdots + a_{n-1} + a_n$$

Exercise 1. Write the sum in expanded form

1. $\displaystyle\sum_{i=1}^{6} i(i+1)$.

2. $\displaystyle\sum_{i=4}^{9} (-1)^i(3i+1)$.

3. $\displaystyle\sum_{j=n}^{n+4} (j+2)$.

4. $\displaystyle\sum_{i=1}^{n} f(x_i)\Delta x_i$

Exercise 2. Write in sigma notation

1. $\sqrt{3} + \sqrt{4} + \sqrt{5} + \sqrt{6} + \sqrt{7} + \sqrt{8} + \sqrt{9} + \sqrt{10}$.

2. $\frac{3}{7}+\frac{4}{8}+\frac{5}{9}+\frac{6}{10}+\cdots+\frac{22}{26}+\frac{23}{27}+\frac{24}{28}$.

3. $1-x+x^2-x^3+x^4-\cdots+(-1)^n x^n$.

Theorem: Let c and d be two constants with $d \neq 1$ and n a positive integer, then

1. $\sum_{i=1}^{n} 1 = 1+1+\cdots+1 = n; \quad \sum_{1}^{n} c = c+c+\cdots+c = cn.$
2. $\sum_{i=1}^{n} i = 1+2+3+4\cdots+n = \frac{n(n+1)}{2}$
3. $\sum_{i=1}^{n} i^2 = 1+2^2+3^2+4^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$
4. $\sum_{i=0}^{n} d^i = d^0+d^1+d^2+\cdots+d^n = \frac{1-d^{n+1}}{1-d}$

Exercise 3. Evaluate the following sums

1. $\sum_{j=1}^{8}(3j-2)$.

2. $\sum_{k=1}^{10}\cos(k\pi/2)$.

3. $\sum_{i=1}^{n}(i+1)(i+3)$.