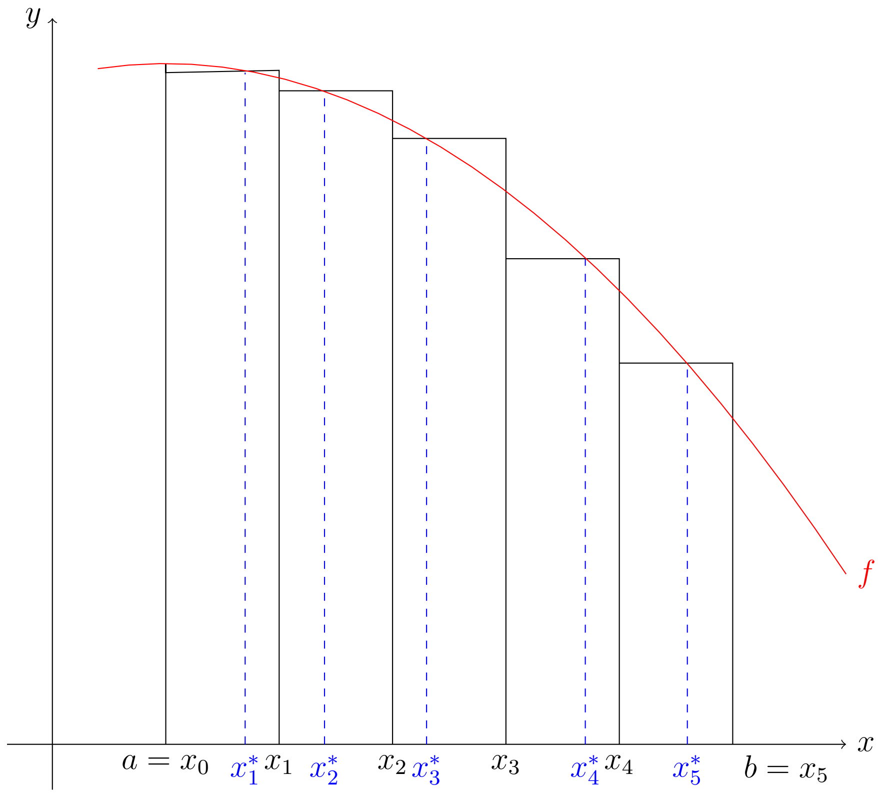
Section 6.2,6.3

Exercise 1. Using the rectangles, find an approximation of the area between $x = a$, $x = b$ $y = 0$ and $y = f(x)$.

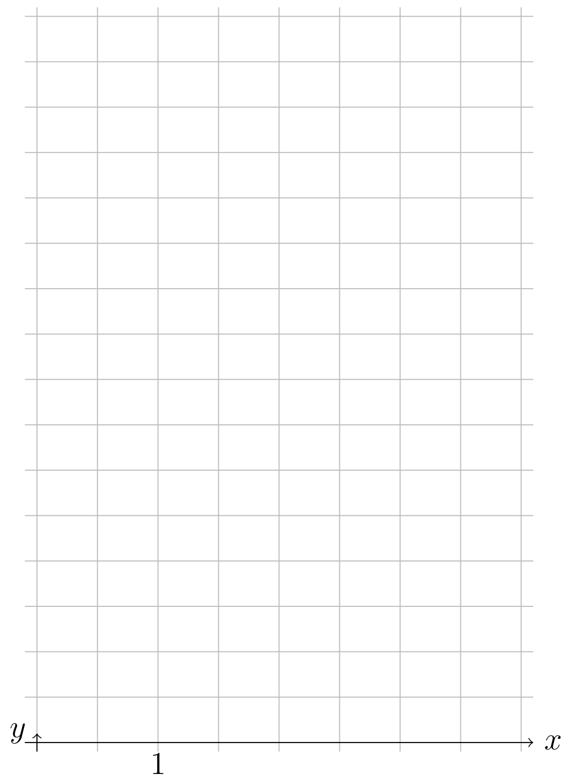


Exercise 2. Given the function $f(x) = 16 - x^2$, the interval $[0, 4]$ the partition points $\{0, 1, 2, 3, 4\}$, $x_i^* =$ midpoint within the ith subinterval.

1. Find $||P||$.

2. Find the sum of the areas of the approximating rectangles.

3. Sketch the graph of f and the approximating rectangles.

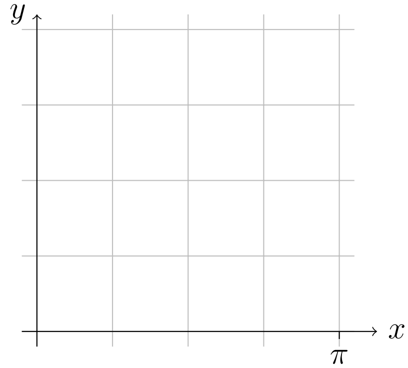


Exercise 3. You are given the function $f(x) = 4\cos x$ on the interval $[0, \pi/2]$, the partition points $\{0, \pi/6, \pi/4, \pi/3, \pi/2\}$, x_i^* =left point within the ith subinterval.

1. Find $||P||$.

2. Find the sum of the areas of the approximating rectangles.

3. Sketch the graph of f and the approximating rectangles.



Theorem: Given a continuous positive function f on $[a, b]$, The area A between $x = a$, $x = b$, $y = 0$ and $y = f(x)$ is

$$A = \lim_{||P|| \to 0} \sum_{i=1}^{n} f(x_i^*) \Delta x_i$$

Exercise 4. find the area under the curve $y = x^2 + 2$ from $a = 1$ to $b = 4$. Use equal subintervals and take x_i^* to be the right endpoint of the ith subinterval. Sketch the region.

Exercise 5. Determine a region whose area is equal to the limit

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{\pi}{4n}\tan\left(\frac{i\pi}{4n}\right)$$

Definition: If f is a function defined on a closed interval $[a,b]$, let P be a partion of $[a,b]$, with partition points $a = x_0 < x_1 < x_2 < \cdots < x_n = b$. Choose points x_i^* in $[x_{i-1}, x_i]$ and let $\Delta x_i = x_i - x_{i-1}$ and $||P|| = \max\{\Delta x_i\}$, then the definite integral of f from a to b is

$$\int_a^b f(x)\mathrm{d}x = \lim_{||P||\to 0}\sum_{i=1}^{n} f(x_i^*)\Delta x_i.$$

if the limit exists.
If the definite integral exists, f is called integrable on the interval $[a,b]$.

Remark:

Exercise 6. Evaluate each integral by interpreting it in terms of area

1. $\int_1^5 (3x-6)\mathrm{d}x$

2. $\int_{-1}^{2} |x| + 2\mathrm{d}x$

3. $\int_{-2}^{2} \sqrt{4-x^2}\mathrm{d}x$

Theorem:

1. If f is continuous, or increasing, or decreasing on $[a, b]$, f is integrable.

2. If $a > b$,
$$\int_a^b f(x)\mathrm{d}x = -\int_b^a f(x)\mathrm{d}x$$
If $a = b$,
$$\int_a^a f(x)\mathrm{d}x = 0.$$

3. If f is integrable
$$\int_a^b f(x)\mathrm{d}x = \lim_{n\to\infty} \frac{b-a}{n} \sum_{i=1}^{n} f\left(a + i\frac{b-a}{n}\right)$$

4. $\int_a^b c\mathrm{d}x = c(b-a)$.

5. $\int_a^b f(x) + g(x)\mathrm{d}x =$

6. $\int_a^b cf(x)\mathrm{d}x =$

7. $\int_a^b f(x)\mathrm{d}x = \int_a^c f(x)\mathrm{d}x + \int_c^b f(x)\mathrm{d}x$

8. If $f(x) \geqslant 0$ on $[a, b]$, then $\int_a^b f(x)\mathrm{d}x \geqslant 0$.

9. If $m \leqslant f(x) \leqslant M$, for all x, $a \leqslant x \leqslant b$, then
$$m(b-a) \leqslant \int_a^b f(x)\mathrm{d}x \leqslant M(b-a)$$

Exercise 7. Evaluate
$$\int_0^2 x^2\mathrm{d}x$$

Exercise 8. Write

$$\int_1^4 f(x)\mathrm{d}x + \int_4^5 f(x)\mathrm{d}x - \int_2^5 f(x)\mathrm{d}x$$

using a single integral.

Exercise 9. Find an estimation of

$$\int_{\pi/6}^{\pi/3} \cos x \mathrm{d}x$$

Exercise 10. Prove that

$$0 \leqslant \int_0^{\pi/2} x \sin x \mathrm{d}x \leqslant \frac{\pi^2}{8}$$

Exercise 11. Prove that

$$2 \leqslant \int_{-1}^{1} \sqrt{1+x}\mathrm{d}x \leqslant 2\sqrt{2}$$

The fundamental theorem of calculus: If f is continuous on $[a, b]$, and F and antiderivative of f, then

$$\int_a^b f(x)\mathrm{d}x = F(b) - F(a)$$

Exercise 12. Find $\int_0^2 (1+x^2)\mathrm{d}x$

$$\int_1^{\mathrm{e}} \frac{x^2+x+1}{x}\mathrm{d}x$$

$$\int_0^{16} \sqrt[4]{x^3}\mathrm{d}x$$

$$\int_{-1}^{2} (x - 2\,|x|)\mathrm{d}x$$

$$\int_0^{\pi/2} \cos x \mathrm{d}x$$