Texas A&M University

Section 1.1 Vectors

1 Vectors

Definition: A vector is a quantity that has both a direction and a magnitude (length).

Example:

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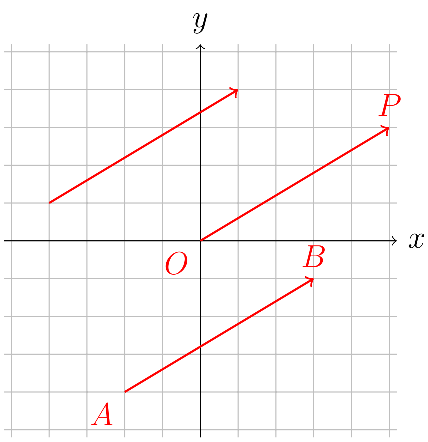
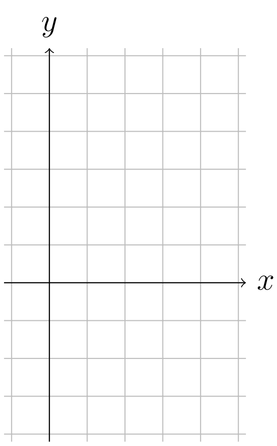


Exercise 1. Let

$$A(1,2), \quad B(3,5), \quad C(4,1).$$



- Find the components of $\overrightarrow{AB}$.

- Draw the position vector of $\overrightarrow{AB}$.

- Find the magnitude of $\overrightarrow{AB}$.

- Find the component of $\overrightarrow{BC}$.

- Draw the position vector of $\overrightarrow{BC}$.

- Find the magnitude of $\overrightarrow{BC}$.

Theorem: Let $A(x_A, y_A)$ and $B(x_B, y_B)$ be two points and $\overrightarrow{u} = \langle x_u, y_u \rangle$ be a vector.

- The components of $\overrightarrow{AB}$ are

$$\overrightarrow{AB}\langle x_B - x_A, y_B - y_A \rangle.$$

- The magnitude of the vector $\overrightarrow{u}$, written $||\overrightarrow{u}||$ or $|\overrightarrow{u}|$, is

$$||\overrightarrow{u}|| = \sqrt{x_u^2 + y_u^2}.$$

- The magnitude of the position vector $\overrightarrow{OA}$ is

$$\left|\left|\overrightarrow{OA}\right|\right| = \sqrt{(x_A)^2 + (y_A)^2}.$$

- The magnitude of the vector $\overrightarrow{AB}$ is

$$\left|\left|\overrightarrow{AB}\right|\right| = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}.$$

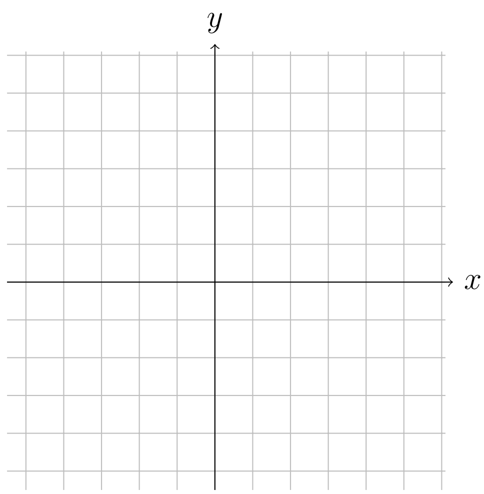
2 Operations on vectors

Definition: Let $\overrightarrow{a}\langle x_a, y_b\rangle$ and $\overrightarrow{b}\langle x_b, y_b\rangle$ be two vectors, and c a real number,
Then

- $\overrightarrow{a} + \overrightarrow{b}$ is the vector with coordinates $\langle x_a + x_b, y_a + y_b\rangle$.
- $c\overrightarrow{a}$ is the vector with coordinates $\langle cx_a, cy_b\rangle$.

Exercise 2. let $\overrightarrow{a} = \langle 2, 3\rangle$, and $\overrightarrow{b} = \langle 1, 2\rangle$ be two vectors.
Plot the vectors

$$\overrightarrow{u_1} = \overrightarrow{a} + \overrightarrow{b}, \quad \overrightarrow{u_2} = \overrightarrow{a} - \overrightarrow{b}, \quad \overrightarrow{u_3} = -\overrightarrow{b}, \quad \overrightarrow{u_4} = 2\overrightarrow{b}, \quad \overrightarrow{u_5} = \frac{1}{2}\overrightarrow{a}, \quad \overrightarrow{u_6} = 2\overrightarrow{a} - 3\overrightarrow{b}$$



Theorem: The $\overrightarrow{a} \neq \overrightarrow{0}$ be a vector. Any scalar multiple vector $c\overrightarrow{a}$ $(c \neq 0)$

- has the same direction as $\overrightarrow{a}$ if $c > 0$.
- has the opposite direction as $\overrightarrow{a}$ if $c < 0$.
- The magnitude $||c\overrightarrow{a}|| = |c|\,||\overrightarrow{a}||$.

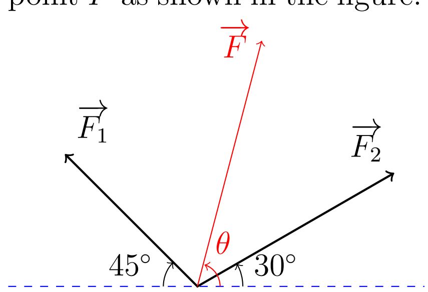
Definition:

$$\mathbf{i} = \langle 1, 0\rangle, \qquad \mathbf{j} = \langle 0, 1\rangle$$

Exercise 3. Find a unit vector that has the same direction as $\overrightarrow{a} = \langle 2, 3\rangle$.

Exercise 4. Let $\overrightarrow{u} = 3\mathbf{i} + 4\mathbf{j}$.
Find a vector $\overrightarrow{v}$ such that $\overrightarrow{u}$ and $\overrightarrow{v}$ have opposite directions, and $||\overrightarrow{v}|| = 2$.

Exercise 5. (27p54) Two forces $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ with magnitude 10lb and 12lb act on an object at a point P as shown in the figure. Find the resultant force $\overrightarrow{F}$ acting at P as well as its magnitude.



Exercise 6. (30p54) Ropes 3m and 5m in length are fastened to a holiday decoration that is suspended over a town square. The decoration has a mass of 5 kg. The ropes fastened at different heights, make angles 52° and 40° with the horizontal. Find the tension in each wire and the magnitude of the tension. (Note: gravity exerts a downward force of 5(9.8)=49N on a 5-kg mass.)