Section 1.2 Dot Product

Exercise 1. (Past Exam) A 30lb block on wheels is at the top of a 20 foot long ramp which is inclined at an angle of 60° above the horizontal. Assuming no friction, find the work done by gravity if the block slides all the way down the ramp.

Definition: The work done by a constant force F in moving an object through a direction d is

$$W = Fd$$

when $\overrightarrow{F}$ is directed along the line of motion.
If the force $\overrightarrow{F}$ makes an angle θ with the line of motion

$$W = d\left|\left|\overrightarrow{F}\right|\right| cos\theta$$

Units: d in ft, F in lb and W in ft-lb.
or d in m, F in N and W in J (joule).

Remark:

Definition: Let $\overrightarrow{a}$ and $\overrightarrow{b}$ be 2 non zero vectors.

$$\overrightarrow{a} \cdot \overrightarrow{b} = |\overrightarrow{a}|\left|\overrightarrow{b}\right| \cos\theta.$$

where θ is the angle between $\overrightarrow{a}$ and $\overrightarrow{b}$.
If $\overrightarrow{a}$ or $\overrightarrow{b}$ is $\overrightarrow{0}$, then $\overrightarrow{a} \cdot \overrightarrow{b} = 0$.

Exercise 2. (Spring 2012) Given a square ABCD with side length 4. If $\overrightarrow{u} = \overrightarrow{AB}$ and $\overrightarrow{v} = \overrightarrow{AC}$, what is $\overrightarrow{u} \cdot \overrightarrow{v}$?

> **Theorem:** Let $\overrightarrow{a} = (x_a, y_a)$ and $\overrightarrow{b} = (x_b, y_b)$ be 2 vectors,
>
> $$\overrightarrow{a} \cdot \overrightarrow{b} = x_a x_b + y_a y_b.$$

Exercise 3. (Spring 2012) If $\overrightarrow{a} = -2\mathbf{i} - \mathbf{j}$ and $\overrightarrow{b} = -4\mathbf{i} + 5\mathbf{j}$, what is the cosine of the angle between $\overrightarrow{a}$ and $\overrightarrow{b}$?

Exercise 4. (Fall 2011) Find the angle between the vectors $\overrightarrow{v} = \langle 1, 2 \rangle$ and $\overrightarrow{w} = \langle 3, 1 \rangle$.

> **Theorem:** Let $\overrightarrow{a}$ and $\overrightarrow{b}$ two non zero vectors,
>
> - $\overrightarrow{a}$ and $\overrightarrow{b}$ are perpendicular iff $\overrightarrow{a} \cdot \overrightarrow{b} = 0$.
> - $\overrightarrow{a}$ and $\overrightarrow{b}$ are parallel iff $\overrightarrow{a} \cdot \overrightarrow{b} = \pm \left|\left|\overrightarrow{a}\right|\right| \cdot \left|\left|\overrightarrow{b}\right|\right|$.

Exercise 5. (21p61) Determine whether $\overrightarrow{a} = \langle 2, -4 \rangle$ and $\overrightarrow{b} = \langle -1, 2 \rangle$ are perpendicular, parallel or neither.

Exercise 6. Find the value of x such that the vectors $\overrightarrow{a} = \langle x, x \rangle$ and $\langle 1, x \rangle$ are orthogonal.

Definition: Given a nonzero vector $\overrightarrow{a} = \langle a_1, a_2 \rangle$, the orthogonal complement of $\overrightarrow{a}$ is the vector $\overrightarrow{a}^{\perp} = \langle -a_2, a_1 \rangle$.

Remark:

Theorem: Let $\overrightarrow{a}$ and $\overrightarrow{b}$ be 2 non zero vectors,
The scalar projection of $\overrightarrow{b}$ onto $\overrightarrow{a}$ is

$$comp_{\overrightarrow{a}} \overrightarrow{b} = \frac{\overrightarrow{a} \cdot \overrightarrow{b}}{|\overrightarrow{a}|}$$

The vector projection of $\overrightarrow{b}$ onto $\overrightarrow{a}$ is

$$\overrightarrow{comp}_{\overrightarrow{a}} \overrightarrow{b} = \left(\frac{\overrightarrow{a} \cdot \overrightarrow{b}}{|\overrightarrow{a}|} \right) \frac{\overrightarrow{a}}{|\overrightarrow{a}|}.$$

Exercise 7. (Spring 2012) Given $\overrightarrow{a} = -4\mathbf{i} + 5\mathbf{j}$ and $\overrightarrow{b} = \mathbf{i} - 2\mathbf{j}$, find the scalar projection of $\overrightarrow{b}$ onto $\overrightarrow{a}$.

Exercise 8. (44p61) Find the distance from the point $(0, 3)$ to the line $y = -3x$.