Section 1.3 Vector valued Function

Exercise 1. The position of a balloon at the time t is determined by

$$x(t) = 2t + 1 \quad \text{and} \quad y(t) = 5t - t^2.$$

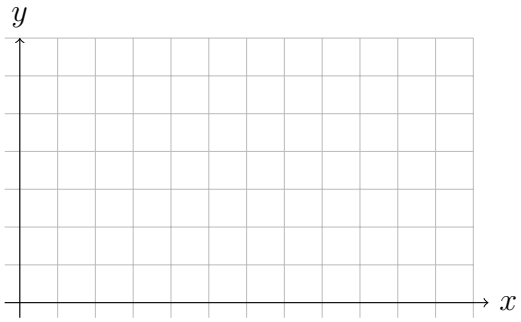
1. What is the position of the balloon at $t = 0$? $t = 2$?

2. At what time is the balloon at the position (7,6)?

3. Will the balloon ever pass through the point $(5, 5)$?

4. At what time will the balloon crash on the floor?

5. sketch the tragectory of the balloon.



Definition: A parametric equation for a curve is in the form $(x(t), y(t))$,
A vector equation for a curve is the data $\overrightarrow{r}(t) = \langle x(t), y(t)\rangle$.
A Cartesian equation is in the form $y = f(x)$ or $x = g(y)$ for some functions $x(t)$, $y(t)$, $f(x)$, and $g(y)$.

Exercise 2. Find a Cartesian equation for the graph of the parametric curve

$$x(t) = -1 + t \quad \text{and} \quad y(t) = t^2 - t.$$

Theorem: A vector equation for the line passing through P and parallel to the vector $\overrightarrow{v}$ is in the form

$$\overrightarrow{r}(t) = \overrightarrow{r_0} + t\overrightarrow{v}$$

where $\overrightarrow{OP} = \overrightarrow{r}_0$.
A parametric equation for the line passing through the point $P(x_0, y_0)$ and parallel to the vector $\overrightarrow{v} = \langle a, b\rangle$ is

$$x(t) = x_0 + ta, \quad y(t) = y_0 + bt.$$

Exercise 3. (Spring 2012) Find a vector equation for the line passing through the point $(-4, 8)$ and parallel to the vector $7\mathbf{i} - 11\mathbf{j}$.

Exercise 4. Find a Cartesian equation, a parametric equation, an a vector equation of the line passing through the point (1, 3) and orthogonal to the vector $\langle 2, 5\rangle$.

Exercise 5. (32p68) Determine whether the line L_1 and L_2 given below are parallel, perpendicular or neither

$$L_1 : \overrightarrow{r}(t) = (-4 + 2t)\mathbf{i} + (5 + t)\mathbf{j} \quad \text{and} \quad L_2 : \overrightarrow{r}(t) = (2 + 3t)\mathbf{i} + (4 - 6t)\mathbf{j}.$$

Exercise 6. Find the coordinates of the intersection point of the line L_1 and L_2 given below

$$L_1 : x(t) = -1 - 2t, y(t) = 2 + t \quad \text{and} \quad L_2 : x(w) = 15 - 3w, y(w) = 3 + 6w$$

Exercise 7. Describe the curve $\overrightarrow{r}(t) = (3 + \sin t)\mathbf{i} + (-2 + \cos t)\mathbf{j}$.