Texas A&M University

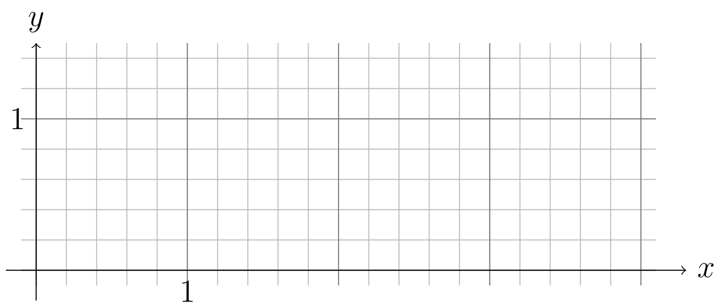
Section 2.2 Introduction to limits

Exercise 1. Let $f(x) = \dfrac{\sqrt{x}-1}{x-1}$.

1. Find the domain of f.

2. Using a table of values of $f(x)$ graph precisely the graph of f for x close to 1.

x										
$f(x)$										



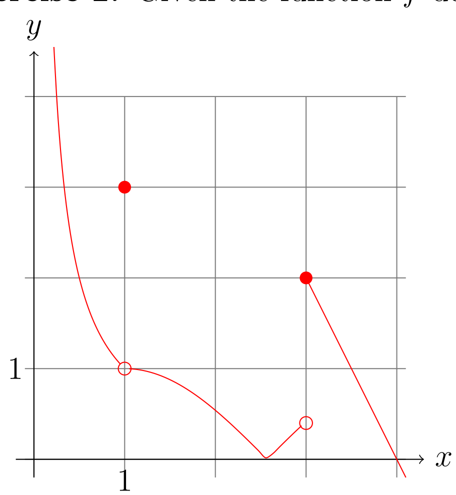
Definition of limits: We write

$$\lim_{x\to c} f(x) = L \quad \text{or} \quad f(x) \longrightarrow L \text{ as } x \to c$$

if the functional value $f(x)$ can be made arbitrarily close to a single real number L when x is close to, but not equal to, c.

Remark:

Exercise 2. Given the function f defined by its graph



Definition: One sided limits: We write

$$\lim_{x\to c^-} f(x) = L$$

the limit from the left or the left hand limit.

Remark: Similar definition for the right-hand limit or limit from the right.

$$\lim_{x\to 4} f(x) = \qquad f(4) =$$

$$\lim_{x\to 1^-} f(x) = \qquad \lim_{x\to 1^+} f(x) =$$

$$\lim_{x\to 1} f(x) = \qquad f(1) =$$

$$\lim_{x\to 3^-} f(x) = \qquad \lim_{x\to 3^+} f(x) =$$

$$\lim_{x\to 3} f(x) = \qquad f(3) =$$

$$\lim_{x\to 0^+} f(x) = \qquad f(0) =$$

Theorem:

$$\lim_{x\to c} f(x) = L$$

if and only if

$$\lim_{x\to c^-} f(x) = L \quad \textbf{and} \quad \lim_{x\to c^+} f(x) = L$$

Exercise 3. Let f be the function

$$\begin{cases} f(x) = x^2 + 5 & \text{if} \quad x < 0 \\ f(x) = 2x + 1 & \text{if} \quad x \geqslant 0 \end{cases}$$

Sketch the graph of f and find

$$f(0) = \qquad \lim_{x\to 0^-} f(x) =$$

$$\lim_{x\to 0^+} f(x) = \qquad \lim_{x\to 0} f(x) =$$

Definition: $y = c$ is a vertical asymptote of the curve $y = f(x)$ if at least one of these is true

$$\lim_{x\to c} f(x) = \pm\infty \qquad \text{or} \qquad \lim_{x\to c^{\pm}} f(x) = \pm\infty.$$

Exercise 4. Determine the infinite limit

1. $\displaystyle\lim_{x\to 3^+} \frac{2}{x-3} =$

2. $\displaystyle\lim_{x\to 3^-} \frac{2}{x-3} =$

3. $\displaystyle\lim_{x\to 3} \frac{2}{x-3} =$

4. $\displaystyle\lim_{x\to 1^+} \frac{3}{(x-1)^2} =$

5. $\displaystyle\lim_{x\to 1^-} \frac{3}{(x-1)^2} =$

6. $\displaystyle\lim_{x\to 1} \frac{3}{(x-1)^2} =$