Section 2.3 Calculating limits

Theorem: Let c be a constant real number. Let f and g be two functions defined in a neighborhood of a such

$$\lim_{x\to a} f(x) = L \quad \text{and} \quad \lim_{x\to a} g(x) = M.$$

1. Sum law: $\lim_{x\to a}(f(x) + g(x)) = L + M.$
2. Difference law: $\lim_{x\to a}(f(x) - g(x)) = L - M.$
3. Constant multiple law: $\lim_{x\to a} cf(x) = c \cdot L.$
4. Product law: $\lim_{x\to a} f(x) * g(x) = L \cdot M.$
5. Quotient law: Assume that $M \neq 0$, then $\lim_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{L}{M}.$
6. Power law: Let n be a positive real number,
$$\lim_{x\to a} f(x)^n = L^n.$$
7. Root law: Let n be a positive integer,
$$\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{L}.$$

Exercise 1. Given 3 functions f, g, and h such that

$$lim_{x\to 1} f(x) = 4, \quad \lim_{x\to 1} g(x) = 3 \quad \lim_{x\to 1} h(x) = -2.$$

Find the limit

$$\lim_{x\to 1} \frac{\sqrt{f(x)} - 2h(x)}{g(x) + h(x)}$$

Exercise 2. Find the limit $\lim_{x\to 2} \dfrac{2x+1}{x^2-4x+3}$.

Theorem: Let f be a polynomial function or a rational function.Let a be in the domain of f, then

$$\lim_{x\to a} f(x) = f(a).$$

Exercise 3. Find $\lim_{x\to 1} \dfrac{(x-1)^2-1}{x-1}$.

Exercise 4. Find the limits $\lim_{x\to 0} |x|$ and $\lim_{x\to 0} sign(x)$.

Theorem:

$$\lim_{x\to a} f(x) = L$$

if and only if

$$\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = L.$$

Exercise 5. Find the limit $\lim_{x\to 0} x^2 \cos\frac{1}{x}$.

The Squeeze Theorem: If

$$f(x) \leqslant g(x) \leqslant h(x)$$

for all x in an interval containing a (except possibly at a),
If

$$\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$$

then $\lim_{x\to a} g(x)$ exists and equals L.

Exercise 6. Find $\lim_{x\to -2} \dfrac{x^2-2x-8}{x+2}$.

Exercise 7. Find $\lim_{x\to 1} \dfrac{\sqrt{x+2}-\sqrt{3}}{x-1}$.

$\lim_{x\to c} f(x)$	$\lim_{x\to c} g(x)$	$\lim_{x\to c}(f(x)+g(x))$	$\lim_{x\to c}(f(x)-g(x))$	$\lim_{x\to c}(f(x)g(x))$	$\lim_{x\to c}\frac{f(x)}{g(x)}$
0	0	0	0	0	*indeterminate*
k	0^+	k	k	0	$sign(k)\infty$
∞	0^+	∞	∞	*indeterminate*	∞
∞	∞	∞	*indeterminate*	∞	*indeterminate*
∞	$-\infty$	*indeterminate*	∞	$-\infty$	*indeterminate*
0	∞	∞	$-\infty$	*indeterminate*	0