Section 2.5 Continuity

Definition: let f be a function defined over an open interval I. Let a be a point of I.
We say that f is continuous at $x = a$ if $f(a) = \lim_{x \to a} f(x)$.
We say that f is continuous on the interval I if f is continuous at any points of I.

Remark:

Theorem: The functions

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are continuous on their domain.

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Exercise 1. Is the function

$$f(x) = \begin{cases} \dfrac{x^2 - 3x + 2}{x - 2}, & \text{if } x \neq 2 \\ 3, & \text{if } x = 2 \end{cases}$$

continuous on $\mathbb{R}$?

Exercise 2. Why is the function $g(x) = \sqrt{\dfrac{2x-1}{x-3}}$ continuous on its domain? State the domain.

Exercise 3. Is $h(x) = |x^3 - x|$ continuous on $\mathbb{R}$? Why?

Exercise 4. Find the point at which k is discontinuous. Sketch the graph of f.

$$k(x) = \begin{cases} 2x+1 & \text{if } x \leqslant -1 \\ 3x & \text{if } -1 < x < 1 \\ 4x-1 & \text{if } x \geqslant 1 \end{cases}$$

Exercise 5. Find the value of c such that the function

$$l(x) = \begin{cases} cx-3 & \text{if } x \leqslant 2 \\ x^2-c & \text{if } 2 < x \end{cases}$$

Exercise 6. Show that the function

$$m(x) = \frac{2-\sqrt{x}}{4-x}$$

has a removable discontinuity at $x = 4$.

> **The intermediate value Theorem:** Suppose that f is continuous on the closed interval $[a, b]$. Let M a number between $f(a)$ and $f(b)$, Then there exists a number c such that $f(c) = M$.

Exercise 7. Prove that the following equations have a solution in the given interval:

1. $x^5 - 3x^2 + 1 = 0$ on $[0, 1]$

2. (48p121) $x^2 = \sqrt{x+1}$ on $[1, 2]$.