Section 2.6 Limits at infinity; Horizontal asymptote

Exercise 1. Complete the following table

x	10	100	1000	10000
x^3				
$\dfrac{x+1}{x-1}$				

Definition: We say $\lim_{x\to+\infty} f(x) = L$ if $f(x)$ approaches L when x increases without bound.
$\lim_{x\to-\infty} f(x) = L$ if $f(x)$ approaches L when x decreases without bound.

Definition: The line $y = L$ is a horizontal asymptote to the graph of f if $\lim_{x\to+\infty} f(x) = L$ or $\lim_{x\to-\infty} f(x) = L$.

Limits of Power Functions at Infinity: If p is a positive real number and k any non zero real then

- $\lim_{x\to+\infty} \dfrac{k}{x^p} = 0$
- $\lim_{x\to-\infty} \dfrac{k}{x^p} = 0$
- $\lim_{x\to+\infty} kx^p = \pm\infty$ depending on the sign of k.
- $\lim_{x\to-\infty} kx^p = \pm\infty$ depending on the sign of k.

Exercise 2. Find the limit of $p(x) = x^5 - 23x^4 + 12x^3 + x - 2$ at $+\infty$ and at $-\infty$.

Exercise 3. Find the horizontal asymptotes for following functions :

- $f_1(x) = \sqrt{x+3} - \sqrt{x-5}$.

- $f_2(x) = \dfrac{x-3}{2x+5}$.

- $f_3(x) = \dfrac{x^9 - 7x^5 + 2}{x^9 - 3x}$.

- $f_4(x) = \dfrac{x^2 + 2}{x^3 - 3x}$.

- $f_5(x) = \dfrac{x-2}{\sqrt{9x^2 - 4x + 7}}$.

Theorem: Let P be a polynomial function and f a rational function,
the limit of P at $\pm\infty$ is the limit of the monomial of highest degree.
The limit of f at $\pm\infty$ is the limit of the quotient: monomial of highest degree of the numerator divided by the monomial of highest degree of the denominator.