University of Washington

Section 2.1. Linear differential equation, Method of integrating factors

Remark: A linear differential equation of order 1 can be written in the form

$$y' + p(x)y = g(x).$$

Exercise 1.

1. Solve
$$\frac{\mathrm{d}y}{\mathrm{d}x} = e^{3x}$$

2. Solve the differential equation
$$x^2y' + 2xy = e^{3x}$$

Exercise 2.

- Find the derivative of $y(x)\,\mathrm{e}^{5x}$

- Solve
$$y' + 5y = 1 + \mathrm{e}^{x}$$

Method of integrating factors:

- Write the equation in the standard form
(coefficient of y' is **1**.)

$$\frac{\mathrm{d}y}{\mathrm{d}x} + P(x)y = g(x)$$

- Calculate the integrating factor $\mu(x)$ defined by the formula

$$\mu(x) = \exp\left(\int P(x)\mathrm{d}x\right)$$

- Multiply the equation in standard form by $\mu(x)$.

$$\mu(x)\frac{\mathrm{d}y}{\mathrm{d}x} + \underbrace{P(x)\mu(x)}_{\frac{\mathrm{d}\mu}{\mathrm{d}x}}y = \mu(x)g(x)$$

- Using the fact that the left-hand side of the equation is $\frac{\mathrm{d}(\mu(x)y)}{\mathrm{d}x}$, integrate the equation

- Solve for y by dividing by $\mu(x)$ to obtain the general solution.

Exercise 3. Solve

$$y' - y\tan x = \sin x\cos x$$

Exercise 4. Solve the initial value problem

$$ty' + (t+1)y = t \quad y(\ln(2)) = 1, \quad t > 0$$

Exercise 5. Find the value of y_0 for which the solution of the initial value problem

$$y' - y = 1 + 3\sin t, \quad y(0) = y_0$$

remains finite as $t \to \infty$.