University of Washington

Sections 2.7 Numerical approximation: Euler's method

Exercise 1. Given the initial value problem

$$y' = 3y - 2t, y(1) = 2$$

1. Trace the tangent to the solutions that pass through the point $(0, 1)$, $(1, 1)$, $(3, 4)$, $(1, 2)$..

> **Definition:** A direction field is a plot of short line segments drawn at various points in the xy-plane showing the direction of the solution curves.

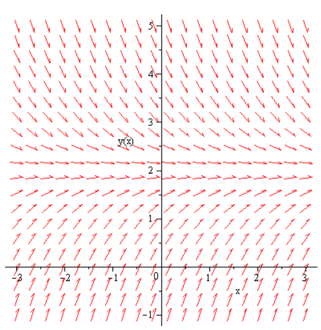

Figure 1: Direction field for the differential equation $y' = 2 - y$

2. Find an approximate value of the solution to the initial value problem

$$y' = 3y - 2t, \qquad y(1) = 0$$

at $t = 1$ using Euler method with $h = 0.2$.

3. Find the exact solution to the initial value problem and evaluate the value of the solution at $t = 1$.

Euler's Method: Let

$$y' = f(t, y), \quad y(t_0) = y_0$$

be an initial value problem for a first order differential equation. Let h be a positive number called step size.
The sequence of points (t_n, y_n) for the Euler's method are defined by the recursive formulae

$$\begin{cases} t_{n+1} = t_n + h \\ y_{n+1} = y_n + hf(t_n, y_n) \end{cases} \quad \text{for } n = 1, 2, \cdots .$$

provide an approximation of the solution to the initial value problem.

Exercise 2. Given the initial value problem

$$y' = t^2 + y^2, \qquad y(0) = 1$$

How can we find the value of y at $t = 0.2$? an approximation of $y(0.2)$?

Runge Kutta method: Let

$$y' = f(t, y), \quad y(t_0) = y_0$$

be an initial value problem for a first order differential equation. Let h be a positive number called step size.
An approximation of the solution to the initial value problem is given by the sequence of points (t_n, y_n) defined by the recursive formulae

$$\begin{cases} t_{n+1} = t_n + h \\ y_{n+1} = y_n + h\dfrac{a_n + 2b_n + 2c_n + d_n}{6} \end{cases} \quad \text{for } n = 1, 2, \cdots .$$

where

$$\begin{cases} a_n = f(t_n, y_n) \\ b_n = f\left(t_n + \dfrac{h}{2}, y_n + \dfrac{ha_n}{2}\right) \\ c_n = f\left(t_n + \dfrac{h}{2}, y_n + \dfrac{hb_n}{2}\right) \\ d_n = f\left(t_n + h, y_n + hc_n\right) \end{cases}$$