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## Review for the Final Exam

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**Exercise 1.** Find the general solution of the given differential equation or initial value problem

1. (1p183)  $y'' - 2y' - 3y = 3e^{3t}$
2. (2p183)  $y'' + 2y' + 5y = 3\sin 2t$ .
3. (17p183)  $y'' + 4y = 3\sin 2t$ ,  $y(0) = 2$ ,  $y'(0) = -1$ .
4. (13p183)  $y'' + y' - 2y = 2t$ ,  $y(0) = 0$ ,  $y'(0) = 1$ .

**Exercise 2.** Use the method of variation of parameters to find a particular solution of

1. (7p189)  $y'' + 4y' + 4y = t^{-2}e^{-2t}$ .
2. (1p189)  $y'' - 5y' + 6y = 2e^t$ .

**Exercise 3.** Determine a suitable form for a particular solution if the method of undetermined coefficient were to be used

1. (21p184)  $y'' + 3y' = 2t^4 + t^2e^{-3t} + \sin 3t$
2. (22p184)  $y'' + y = t(11 + \sin t)$
3. (24p184)  $y'' + 2y' + 2y = 3e^{-t} + 2e^{-t}\cos t + 4e^{-t}t^2\sin t$ .
4. (27p185)  $y'' + 3y' + 2y = e^t(t^2 + 1)\sin 2t + 3e^{-3t}\cos t + 4e^t$ .

**Exercise 4.** Using the definition of the Laplace transform to find the Laplace transform of the following functions

1. (5(a)p315)  $f(t) = t$ ,
2.  $f(t) = e^{2t} + 3$
3. (23p315)  $f(t) = \begin{cases} t & 0 \leq t \leq 1 \\ 1 & 1 < t \end{cases}$

**Exercise 5.** Find the inverse Laplace transform of the function

1. (1p324)  $\frac{3}{s^2 + 4}$ .

2. (6p324)  $\frac{2s - 3}{s^2 - 4}$ .
3. (8p324)  $\frac{8s^2 - 4s + 12}{s(s^2 + 4)}$ .
4. (9p324)  $\frac{1 - 2s}{s^2 + 4s + 5}$ .

**Exercise 6.** Sketch the graph of the given functions on the interval  $t \geq 0$ .

1. (1p333)  $g(t) = u_1(t) + 2u_3(t) - 6u_4(t)$ .
2. (2p333)  $g(t) = (t - 3)u_2(t) - (t - 2)u_3(t)$ .
3. (3p333)  $g(t) = f(t - \pi)u_\pi(t)$  where  $f(t) = t^2$ .

**Exercise 7.** Express  $f(t)$  in terms of unit step function  $u_c(t)$ .

1. (10p333)  $f(t) = \begin{cases} t^2, & 0 \leq t < 2 \\ 1, & 2 \leq t \end{cases}$
2. (12p333)  $f(t) = \begin{cases} t & 0 \leq t < 2 \\ 2 & 2 \leq t < 5 \\ 7 - t & 5 \leq t < 7 \\ 0 & 7 \leq t \end{cases}$

**Exercise 8.** Find the Laplace transform of

1. (14p333)  $f(t) = \begin{cases} 0, & 0 \leq t < 1 \\ t^2 - 2t + 2, & 1 \leq t \end{cases}$
2. (17p333)  $f(t) = (t - 3)u_2(t) - (t - 2)u_3(t)$ .
3. (18p333)  $f(t) = t - u_1(t)(t - 1)$ .
4.  $f(t) = \begin{cases} t, & 0 \leq t < 1 \\ t - 1, & 1 \leq t < 2 \\ 2t - 2 & 2 \leq t < 3 \\ 8 & 3 \leq t \end{cases}$ .

**Answer:**  $f(t) = t - u_1(t) + u_2(t)(t - 1) + u_3(t)(10 - 2t)$

$$\mathcal{L}\{f\}(s) = \frac{1}{s^2} - \frac{e^{-s}}{s} + e^{-2s} \left( \frac{1}{s^2} + \frac{1}{s} \right) + e^{-3s} \left( \frac{4}{s} - \frac{2}{s^2} \right),$$

**Exercise 9.** Find the inverse Laplace transform of

1. (19p333)  $F(s) = \frac{3!}{(s - 2)^4}$ .

2. (20p333)  $F(s) = \frac{e^{-2s}}{s^2 + s - 2}$ .
3. (21p333)  $F(s) = \frac{2(s-1)e^{-2s}}{s^2 - 2s + 2}$ .
4. (4p324)  $F(s) = \frac{3s}{s^2 - s - 6}$ .
5. (8p324)  $F(s) = \frac{8s^2 - 4s + 12}{s(s^2 + 4)}$ .
6. (10p324)  $F(s) = \frac{2s - 3}{s^2 + 2s + 10}$ .
7. (23p333)  $F(s) = \frac{(s-2)e^{-s}}{s^2 - 4s + 3}$ .
8.  $F(s) = \frac{2se^{-3s}}{s^2 - 2s + 2}$ .

**Exercise 10.** Find the solution of the given initial value problem

**Exercise 11.** Use the Laplace transform to solve the initial value problem

1. (11p324)  $y'' - y' - 6y = 0, \quad y(0) = 1, \quad y'(0) = -1$
2. (21p325)  $y'' - 2y' + 2y = \cos t, \quad y(0) = 1, \quad y'(0) = 0$
3. (14p325)  $y'' - 4y' + 4y = 0, \quad y(0) = 1, \quad y'(0) = 1.$
4. (23p325)  $y'' + 2y' + y = 4e^{-t}, \quad y(0) = 2, \quad y'(0) = -1.$
5. (4p340)  $y'' + 4y = \sin t + u_\pi(t) \sin(t - \pi), \quad y(0) = 0, \quad y'(0) = 0.$
6. (2p340)  $y'' + 2y' + 2y = \begin{cases} 1 & \pi \leq t < 2\pi \\ 0 & 0 \leq t < \pi \text{ and } 2\pi \leq t \end{cases}$
7. (6p340)  $y'' + 3y' + 2y = u_2(t) \quad y(0) = 0, \quad y'(0) = 1.$
8. (9p340)  $y'' + y = \begin{cases} \frac{t}{2} & 0 \leq t < 6 \\ 3 & t \geq 6 \end{cases}$
9. (10p340)  $y'' + y' + \frac{5}{4}y = \begin{cases} \sin t & 0 \leq t < \pi \\ 0 & t \geq \pi \end{cases}.$
10. (5p340)  $y'' + 3y' + 2y = f(t), \quad y(0) = 0, \quad y'(0) = 0, \quad f(t) = \begin{cases} 1, & 0 \leq t < 10 \\ 0, & 10 \leq t. \end{cases}$
11. (11p340)  $y'' + 4y = u_\pi(t) - u_{3\pi}(t), \quad y(0) = 0, \quad y'(0) = 0.$

12. (3p340)  $y'' + 4y = \sin t - u_{2\pi}(t) \sin(t - 2\pi), \quad y(0) = 0, \quad y'(0) = 0.$

13. (7p340)  $y'' + y = u_{3\pi}(t) \quad y(0) = 1, \quad y'(0) = 0.$