Sections 2.5 Autonomous Equations

Definition: A first order differential equation in the form

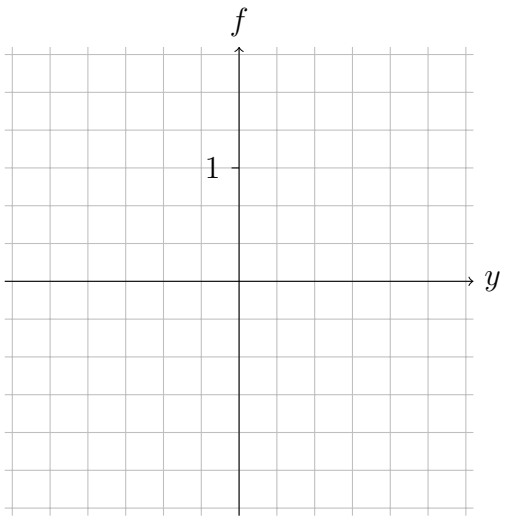
$$\frac{\mathrm{d}y}{\mathrm{d}t} = f(y)$$

is called autonomous – that is the independent variable t does not appear explicitly.

Exercise 1. Given the differential equation

$$y'(t) = y^2(1 - y^2).$$

1. Sketch the graph of f versus y.



2. Find all the constant solutions to the differential equation.

Definition: The constant solutions are called equilibrium solutions. They correspond to the zeros of f.

3. Show that for any real A, if $y_1(t)$ is solution to the problem, $y_2(t) = y(t + A)$ is solution too. Interpret the result geometrically.

4. For which values of y are the solutions $y(t)$ increasing? decreasing? Sketch the phase line for the problem.

Definition: An equilibrium solution is stable if any neighboring solution is attracted to it as t goes to infinity.
An equilibrium solution is unstable if any neighboring solution is repelled by the solution.
An equilibrium solution that is not stable or unstable is called semistable.

5. Find the concavity of the solutions.

6. Sketch the graph of the solutions.