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Main results about Hyperbolic functions

Definition:

$$\cosh(x) = \mathrm{ch}(x) = \frac{e^x + e^{-x}}{2}, \qquad \sinh(x) = \mathrm{sh}(x) = \frac{e^x - e^{-x}}{2}$$

Properties:

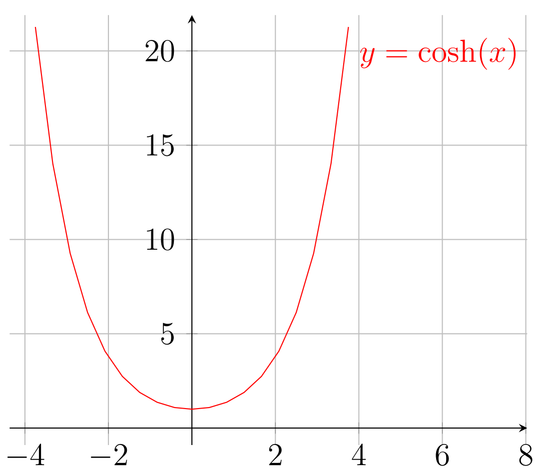
$$\cosh'(x) = \sinh(x), \qquad \sinh'(x) = \cosh(x)$$

$$\cosh(0) = 1, \qquad \sinh(0) = 0$$

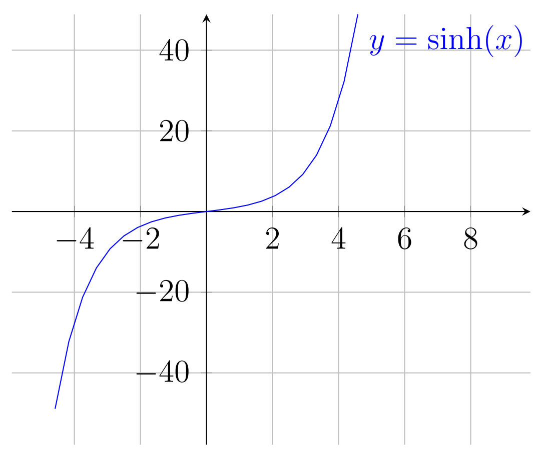
$$\cosh^2 x - \sinh^2 x = 1$$

The function $\cosh(x)$ is an even function.
The function $\sinh(x)$ is an odd function.
Their domain is $\mathbb{R}$.
They are not periodic.

Their graphs:



(a) Graph of $\cosh(x)$



(b) Graph of $\sinh(x)$

Other results:

$$\lim_{x\to 0} \frac{\sinh x}{x} = 1$$

$$\cosh(x) + \sinh(x) = e^x$$

$$\cosh(x) - \sinh(x) = e^{-x}$$

$$\cosh(a+b) = \cosh(a)\cosh(b) + \sinh(a)\sinh(b)$$

$$\cosh(a-b) = \cosh(a)\cosh(b) - \sinh(a)\sinh(b)$$

$$\sinh(a+b) = \sinh(a)\cosh(b) + \sinh(b)\cosh(a)$$

$$\sinh(a-b) = \sinh(a)\cosh(b) - \sinh(b)\cosh(a)$$

$$\cosh(p)\cosh(q) = \frac{1}{2}\left(\cosh(p+q) + \cosh(p-q)\right)$$

$$\cosh(p)\sinh(q) = \frac{1}{2}\left(\sinh(p+q) - \sinh(p-q)\right)$$

$$\sinh(p)\cosh(q) = \frac{1}{2}\left(\sinh(p+q) + \sinh(p-q)\right)$$

$$\sinh(p)\sinh(q) = \frac{1}{2}\left(\cosh(p+q) - \cosh(q-p)\right)$$