

## Homework 1

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Last name: \_\_\_\_\_

First name: \_\_\_\_\_

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*Due in class on Wednesday April 4th.*

**Exercise 1.** Express the given systems in matrix notation:

$$\begin{aligned}x'_1 &= (\cos 2t)x_1 + e^t x_2 \\x'_2 &= (\sin 2t)x_1 + (4 + 2t^3)x_2 \\x'_3 &= (\sin t)x_3\end{aligned}$$

**Exercise 2.** Transform the given differential equation or system into an equivalent system of first order differential equations. Give your answer in matrix notation.

1.  $(1 - t^2)y''' - 2ty'' + 5y = e^t$

$$2. \begin{cases} x'' - 2x' + 5y' + x = 0 \\ y'' + 7y' - 3y + 2x = 0 \end{cases}$$

**Exercise 3.** Find all the solutions to the following systems

$$1. \begin{cases} x_1 + 2x_2 + 4x_3 + x_4 & = 0 \\ -x_1 - 2x_2 - 2x_3 & = 1 \\ -2x_1 - 4x_2 - 8x_3 + 2x_4 & = 4 \\ x_1 + 4x_2 + 2x_3 & = -3 \end{cases} .$$

$$2. \begin{cases} x_1 - x_2 + 2x_3 + 2x_4 & = 2 \\ 2x_1 - 2x_2 - 4x_3 + 3x_4 & = 4 \\ 3x_1 - 3x_2 + 6x_3 + 5x_4 & = 6 \\ 4x_1 - 4x_2 + 8x_3 + 7x_4 & = 8 \end{cases}.$$

**Exercise 4.** Check whether the matrix  $\begin{pmatrix} e^{2t} & e^{3t} \\ -e^{2t} & -2e^{3t} \end{pmatrix}$  is solution to the system  $X' = \begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix} X$ .

**Exercise 5.** Determine whether the following vector functions are linearly independent or linearly dependent:  $X_1(t) = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ ,  $X_2(t) = \begin{pmatrix} t \\ 0 \\ t \end{pmatrix}$ ,  $X_3(t) = \begin{pmatrix} t^2 \\ 0 \\ t^2 \end{pmatrix}$ .

**Exercise 6.** Let  $A = \begin{pmatrix} 2 & 4 \\ 1 & 1 \end{pmatrix}$ . Find  $A^2 - 3A$ .

**Exercise 7.** Find the eigenvalues and eigenvectors of the matrices

1.  $A = \begin{pmatrix} 5 & -3 \\ 2 & 0 \end{pmatrix}$ .

2.  $B = \begin{pmatrix} 3 & -5 & -5 \\ -1 & 2 & 2 \\ 3 & -5 & -5 \end{pmatrix}.$

3.  $C = \begin{pmatrix} 1 & -2 & 2 \\ -2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix}.$