

Homework 2

Last name: _____

First name: _____

Section: _____

*Due in class on **Wednesday** April 13th.*

Give all your answers in terms of **real valued functions**.

Exercise 1. Show that the vector functions

$$X_1(t) = \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}, \quad X_2(t) = \begin{pmatrix} \sin t \\ -\cos t \end{pmatrix}$$

are a fundamental set of solutions of the system

$$X' = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} X$$

Exercise 2. Consider the system

$$X' = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} X + \begin{pmatrix} 0 \\ e^t \end{pmatrix}$$

1. Show that the vector function $X_p = \begin{pmatrix} e^t \\ 0 \end{pmatrix}$ is a particular solution.

2. Find the general solution to the non homogeneous system.

Exercise 3. Find the general solution to

$$X' = \begin{pmatrix} -3 & -4 \\ 2 & 3 \end{pmatrix} X$$

Give the type of the critical point $(0,0)$, determine whether it is stable, asymptotically stable or unstable. Sketch several trajectories in the phase plane.

Find the solution to the initial value problem $X(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

Exercise 4. Find the general solution to the system

$$X' = \begin{pmatrix} 0 & 1 & 1 \\ 2 & 1 & 2 \\ -1 & -1 & -2 \end{pmatrix} X$$

Exercise 5. Find the general solution to the system

$$X' = \begin{pmatrix} 3 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & -5 & -5 \end{pmatrix} X$$