Homework 4

Last name: _____
First name: _____
Section: _____

Due in class on Wednesday May 3rd.

Exercise 1. Determine all the solutions if any of the given boundary value problem

1.
$$y'' + 9y = 0, \qquad y(0) = 0, \quad y'(\pi) = -6$$

2.
$$y'' - y = 1 - 2x, \qquad y(0) = 0, \quad y(1) = 1 + e$$

3.
$$y'' + y = 0, \qquad y(0) = 1, \quad y(2\pi) = 0$$

4.
$$y'' + 4y = 0, \quad y(-\pi) = 0, \quad y(\pi) = 0$$

Exercise 2. Find the eigenvalues r for which the given problem has a non trivial solution and determine a corresponding eigenfunction:

1.
$$y'' + ry = 0, \qquad y'(0) = 0, \quad y'(\pi) = 0$$

2.

$$y'' + ry = 0, \qquad y(0) = y'(0), \quad y(\pi) = y'(\pi)$$

Exercise 3. Use the method of separation of variables to find the solution to the heat flow problem

$$5\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \qquad 0 < x < \pi, \quad t > 0$$
$$u(t,0) = u(t,\pi) = 0, \qquad t > 0$$
$$u(0,x) = \sin x + 3\sin 2x - 5\sin 4x, \qquad 0 < x < \pi$$

Exercise 4. Solve the heat flow problem with insulated boundaries

$$\begin{aligned}
&\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}, && 0 < x < 1, \quad t > 0\\
&\frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(1,t) = 0, && t > 0\\
&u(x,0) = \cos(\pi x) - 2\cos(3\pi x), && 0 < x < 1
\end{aligned}$$