
Homework 5

Last name: _____

First name: _____

Due in class on Friday May 12th.

Exercise 1. Given the Partial Differential Equation

$$\frac{\partial^2 u}{\partial t^2} + \frac{\partial u}{\partial t} + u = \alpha^2 \frac{\partial^2 u}{\partial x^2}.$$

Show that the method of separation of variables $u(t, x) = X(x)T(t)$ yield to the system of ordinary differential equations

$$\begin{cases} T''(t) + T'(t) + (1 - \lambda\alpha^2)T(t) = 0 \\ X''(x) - \lambda X(x) = 0 \end{cases}$$

Exercise 2. Given n and m two natural numbers, evaluate the integral

$$\int_{-L}^L \sin\left(\frac{n\pi x}{L}\right) \sin\left(\frac{m\pi x}{L}\right) dx$$

Exercise 3. Compute the Fourier series of the 2-periodic function defined by

$$f(x) = \begin{cases} 1 & -1 < x < 0 \\ 2, & 0 < x < 1 \end{cases}$$

Exercise 4. Given the 2π -periodic function defined by

$$g(x) = \begin{cases} x & 0 < x < \pi \\ x + \pi, & -\pi < x < 0 \end{cases}$$

1. Sketch the graph of the function g and its Fourier series over 3 periods.
2. Compute its Fourier series.