

Homework 6

Last name: _____

First name: _____

Due in class on Friday May 19th.

Exercise 1. Given the 2π -periodic function defined by $f(x) = |x|$ on $[-\pi, \pi]$.

1. Find the Fourier series of f .

2. Use the Fourier series to evaluate

$$\sum_{k=1}^{\infty} \frac{1}{(2k-1)^2}$$

Exercise 2. Given the function $f(x) = x^2$ for x in $(0, \pi)$.

1. Define an even extension of f that is 2π periodic. Sketch the graph of the even extension and the graph of the cosines Fourier series over 3 periods.

2. Define an odd extension of f that is 2π -periodic. Sketch the graph of the odd extension and the sines series over 3 periods.

Exercise 3. Given the function f defined on the interval $(0, \pi)$ by $f(x) = \cos 2x$.

1. Consider the 2π -periodic, even extension of f and compute the cosine Fourier series of f .

2. Consider the 2π -periodic, odd extension of f and compute the sine Fourier series of f

3. Consider the **2**-periodic even extension of $f(x) = \cos(2x)$ on $[0, 1]$ and compute its cosines Fourier series.

Exercise 4. Given the function f defined on

$$\begin{aligned}\frac{\partial u}{\partial t} &= 3\frac{\partial^2 u}{\partial x^2}, & 0 < x < 1 \\ u(t, 0) = u(t, 1) &= 0, & t > 0 \\ u(0, x) &= x, & 0 < x < 1\end{aligned}$$

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