

Homework 1

Last name: _____

First name: _____

Due in class on Wednesday April 4th.

Exercise 1. Express the given systems in matrix notation:

$$\begin{aligned} x_1' &= (\cos 2t)x_1 + e^t x_2 \\ x_2' &= (\sin 2t)x_1 + (4 + 2t^3)x_2 \\ x_3' &= (\sin t)x_3 \end{aligned}$$

Let $X(t)$ be the vector $X(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}$. Then the system is equivalent to $X'(t) = \begin{pmatrix} \cos 2t & e^t & 0 \\ \sin 2t & 4+2t^3 & 0 \\ 0 & 0 & \sin t \end{pmatrix} X(t)$.

Exercise 2. Transform the given differential equation or system into an equivalent system of first order differential equations. Give your answer in matrix notation.

1. $(1 - t^2)y''' - 2ty'' + 5y = e^t$

Let $x_1(t) = y(t)$, $x_2(t) = y'(t)$, $x_3(t) = y''(t)$.

then $x_1'(t) = y'(t) = x_2(t)$

$x_2'(t) = y''(t) = x_3(t)$

$x_3'(t) = y'''(t) = \frac{2t}{1-t^2} y'' - \frac{5y}{1-t^2} + \frac{e^t}{1-t^2} = \frac{2t}{1-t^2} x_3 - \frac{5x_1}{1-t^2} + \frac{e^t}{1-t^2}$

Let $X(t)$ be the vector function $\begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}$

then $X'(t) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{-5}{1-t^2} & 0 & \frac{2t}{1-t^2} \end{pmatrix} X(t) + \begin{pmatrix} 0 \\ 0 \\ \frac{e^t}{1-t^2} \end{pmatrix}$