University of Washington

Section 10.3

Definition: A function f is said to be piecewise continuous on an interval $[a, b]$ if the interval can be partitioned by a finite number of points $a = x_0 < x_1 < x_2 < \cdots < x_n = b$ such that

- f is continuous on each open interval (x_i, x_{i+1}).
- f has finite limits from the right and finite limits from the left at the end points.

Theorem: Suppose that f and f' are piecewise continuous on the interval [-L,L].
Suppose that f is $2L$-periodic. Then the Fourier series of f is convergent to

- $f(x)$ if the function f is continuous at x.
- $\dfrac{f(x^+) + f(x^-)}{2}$ if f as a discontinuity at x.

  $f(x^+)$ is the limit from the right of f at x.

  $f(x^-)$ is the limit from the left of f at x.

Exercise 1. Find the Fourier series of the 2-periodic function

$$f(x) = \begin{cases} -1 & -1 \leqslant x < 0, \\ 1 & 0 \leqslant x < 1 \end{cases}$$

Sketch the graph of the function to which the Fourier series converges for 3 periods.